Physics

Q.1 The physical parameters P, Q, R, S can be determined by measuring the parameters a, b, c using the relations.

$P = a^2bc$

$Q = ab^2c$

$R = abc^2$

$S = \sqrt{abc}$

If the maximum percentage error in the measurement of p, q, r is $1^o/_o$, $3^o/_o$, $4^o/_o$ respectively. Then what is the maximum percentage error in measurement of S

Option 1:
$1^o/_o$

Option 2:
$2^o/_o$s.

Option 3:
$3^o/_o$

Option 4:
$4^o/_o$

Correct Answer:
$1^o/_o$

Solution:
As, $P = a^2bc$ ——(1)

$Q = ab^2c$ ——(2)

$R = abc^2$ ——(3)

$S = \sqrt{abc} = (abc)^{\frac{1}{2}}$ ——(4)

From (1) , (2) & (3)

We can write $P.Q.R = (abc)^4$

$\Rightarrow abc = (PQR)^{\frac{1}{4}}$

So, $S = (abc)^{\frac{1}{2}}$

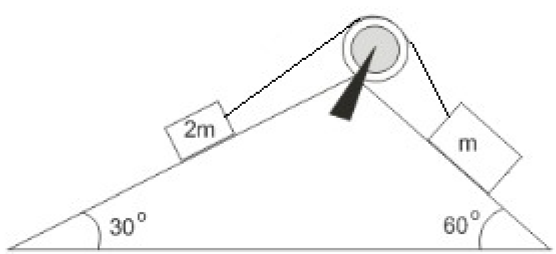
$$\Rightarrow S = (PQR)^{\frac{1}{4}\times\frac{1}{2}} = (PQR)^{\frac{1}{8}}$$

So, $\frac{\Delta S}{S} \times 100 = \frac{1}{8}\left[\frac{\Delta P}{P} \times 100 + \frac{\Delta Q}{Q} \times 100 + \frac{\Delta R}{R} \times 100\right]$

$$\Rightarrow \frac{\Delta S}{S} \times 100 = \frac{1}{8}[1+3+4] = \frac{8}{8} = 1$$

$$^o/_o \; error \; in \; S = 1 \, ^o/_o$$

Q. 2 The acceleration of system of two bodies over a wedge as shown in gure is :-
$(use = g = 10 m/s^2)$



Option 1:
$0.44 m/s^2$

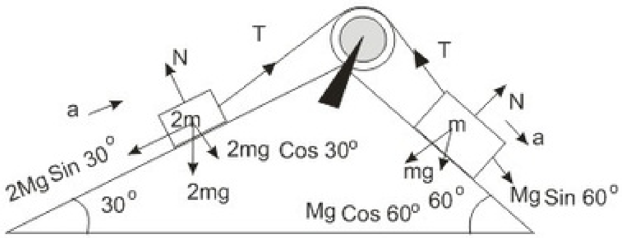
Option 2:
$1.44 \, m/s^2$

Option 3:
$2.7 \, m/s^2$

Option 4:
$0.0175 \, m/s^2$

Correct Answer:
$0.44 m/s^2$

Solution:



Let Tension in string $= T$

Let accleration $= a$.

$$ma = mg\ sin\ 60^\circ - T \_\_\_(1)$$

And $2ma = T - 2\ mg\ sin 30^\circ \_\_\_(2)$

(1) + (2)

$$3\ ma\ = mg\ sin\ 60^\circ - 2\ mg\ sin 30^\circ$$

$$= a = \frac{gsin60^\circ - 2sin30^\circ \times g}{3}$$

$$a = \frac{10 \times \frac{\sqrt{3}}{2} - 2 \times \frac{1}{2} \times 10}{3}$$

$$a = -0.4465\ m/s^2$$

So, our assumed direction of acceleration is wrong, which is indicated by a negative sign.

Now, $|a| = 0.4465\ m/s^2$

Q. 3 The motion of a partice is de ned by a equation :-

$$x = 8 + 9t - t^3$$

Where 'x' is in meter and 'T' is in second. What will be the position (X) of the particle, When its velocity becomes zero:- (use $\sqrt{3} = 1.732$)

Option 1:
$7.52\ m$

Option 2:
$12.8\ m$

Option 3:
$18.4\ m$

Option 4:
$25.19\ m$

Correct Answer:
$18.4\ m$

Solution:

$$x = 8 + 9t - t^3$$

$$v = \frac{dx}{dt} \quad = \quad 9 - 3t^2 = 0$$

$$\Rightarrow \quad t = \sqrt{3}\ second$$

So, $$x = 8 + 9\sqrt{3} - \left(\sqrt{3}\right)^3$$

$$x = 18.39\ m$$

Q. 4 A constant power delivering machine has towed a box, which was initially at rest , along a horizontal straight line. The distance moved by the box in time 't' is proportional to :

Option 1:
$t^{2/3}$

Option 2:
T

Option 3:
$t^{3/2}$

Option 4:
$t^{1/2}$

Correct Answer:
$t^{3/2}$

Solution:

$P = C$

$FV = C$

$M\frac{dV}{dt}V = C$

$\frac{V^2}{2} \propto t$

$V \propto t^{1/2}$

$\frac{dx}{dt} \propto t^{1/2}$

$x \propto t^{3/2}$

Q. 5 A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its centre . Two beads of mass m and negligible size are at the centre of the rod initially. The beads are free to slide along the rod . The angular speed of the system when the beads reach the opposite ends of the rod will be :

Option 1:
$$\frac{M\,\omega_0}{M+m}$$

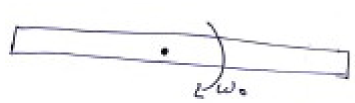
Option 2:
$$\frac{M\,\omega_0}{M+3m}$$

Option 3:
$$\frac{M\,\omega_0}{M+6m}$$

Option 4:
$$\frac{M\,\omega_0}{M+2m}$$

Correct Answer:
$$\frac{M\,\omega_0}{M+6m}$$

Solution:
Angular momentum is conserved only when external torque is zero .



Applying conservation of angular momentum

$$\left(\frac{ML^2}{12}\right)\omega_0 = \left[\frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2\right]\omega^1$$

$$\Rightarrow ML^2\omega_0 = \left(ML^2 + 6mL^2\right)\omega^1$$

$$\Rightarrow \omega^1 = \frac{\omega_0 M}{M + 6m}$$

Q. 6 Two rods A and B of identical dimensions are at temperature 30oC . If A is heated up to 180 oC and B upto ToC , then the new lengths are the same. If the ratio of the coe cients of linear expansion of A and B is 3:2 , then the value of T is :

Option 1:
230

Option 2:
245

Option 3:
255

Option 4:
275

Correct Answer:
255

Solution:

A B

$$T_o = 30 \qquad T_o = 30$$

$$T_f = 180 \qquad T_f = T$$

$$\alpha_A \qquad \alpha_B$$

$$\frac{\alpha_A}{\alpha_B} = \frac{3}{2}$$

$$\triangle l_1 = \triangle l_2$$

$$l\alpha_A \triangle T_A = l\alpha_B \triangle T_B$$

$$\frac{\alpha_A}{\alpha_B} = \frac{\triangle T_B}{\triangle T_A} \Rightarrow \frac{3}{2} = \frac{(T-30)}{(180-30)}$$

$$T - 30 = \frac{3}{2} \times 150 = 225$$

$$T = 30 + 225 = 255$$

$$T = 255°C$$

Q. 7 An 18.10mL sample of gas is at 3.500 atm. The volume (in mL) if the pressure becomes 2.500 atm, with a xed amount of gas and temperature is -

Option 1:
18.1

Option 2:
2.534

Option 3:
1.81

Option 4:
25.34

Solution:
By solving with the help of Boyle's law equation

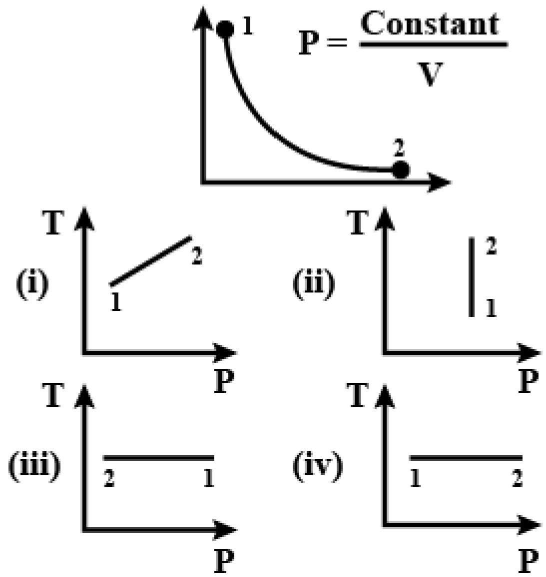
$P_1V_1 = P_2V_2$

$V_2 = P_1V_1/P_2$

$V_2 = (18.10 * 3.500\text{atm})/2.500\text{atm}$

$V_2 = 25.34\text{mL}$

Q. 8 Consider P−V diagram for an ideal gas shown in Figure.



Out of the following diagrams (Figure), which represents the T−P diagram?

Option 1:
iv

Option 2:
ii

Option 3:
iii

Option 4:
i

Correct Answer:
iii

Solution:
In the graph given in question,
PV=constant
And PV=nRT
So, T=constant

So option c is correct

Q. 9 A sound wave of frequency 245 Hz travels with the speed of 300 ms -1 along the positive x-axis. Each point of the wave moves to and fro through a total distance of 6 cm. What will be the mathematical expression of this travelling wave ?

Option 1:
$Y(x, t) = 0.06\left[\sin 5.1x - \left(1.5 \times 10^3\right)t\right]$

Option 2:
$Y(x, t) = 0.03\left[\sin 5.1x - \left(1.5 \times 10^3\right)t\right]$

Option 3:
$Y(x, t) = 0.03\left[\sin 5.1x - \left(0.2 \times 10^3\right)t\right]$

Option 4:
$Y(x, t) = 0.06\left[\sin 0.8x - \left(0.5 \times 10^3\right)t\right]$

Correct Answer:
$Y(x, t) = 0.03\left[\sin 5.1x - \left(1.5 \times 10^3\right)t\right]$

Solution:

$$\omega = 2\pi \mathrm{f}$$
$$= 1.5 \times 10^3$$
$$\mathrm{A} = \frac{6}{2} = 3 \text{ cm} = 0.03 \text{ m}$$

So the equation that suits the following is -

$$Y(x, t) = 0.03\left[\sin 5.1x - \left(1.5 \times 10^3\right)t\right]$$

Q. 10 The electric intensity due to a dipole of length 10 cm and having a charge of 500μC, at a point on the axis at a distance 20 cm from one of the charges in air is:-

Option 1:
6.25×107 N/C

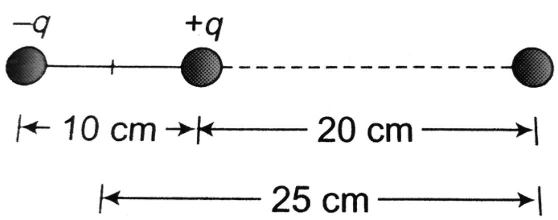
Option 2:
9.28×107 N/C

Option 3:
13.1×1111 N/C

Option 4:
20.5×107 N/C

Correct Answer:
6.25×107 N/C

Solution:



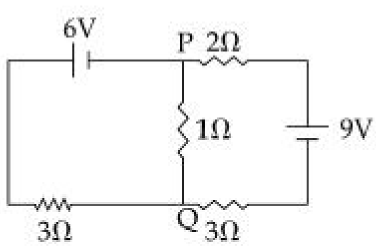
By using $E = 9 \times 10^9 \cdot \frac{2pr}{(r^2 - l^2)^2}$, where

$p = (500 \times 10^{-6}) \times (10 \times 10^{-2}) = 5 \times 10^{-5} c \times m$

$r = 25\text{cm} = 0.25\text{m}, l = 5\text{cm} = 0.05\text{m}$

$$E = \frac{9 \times 10^9 \times 2 \times 5 \times 10^{-5} \times 0.25}{\{(0.25)^2 - (0.05)^2\}^2} = 6.25 \times 10^7 N/C$$

Q. 11



In the circuit shown, the current in the 1Ω resistor is :

Option 1:
1.3 A ,from P to Q

Option 2:
0 A

Option 3:
0.13 A ,from Q to P

Option 4:
0.13 A ,from P to Q

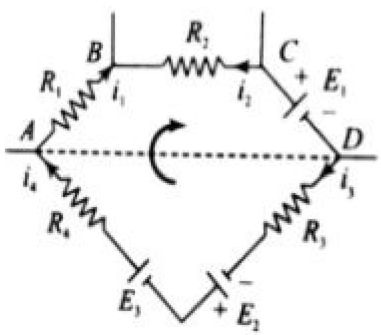
Correct Answer:
0.13 A ,from Q to P

Solution:
As we discussed in

In closed loop -

$$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$$

- wherein



Applying KVL in loop PQCDP

-1I2-3I2+9-2I2+-1I1=0

6I2-I1=9-(i)

Applying KVL in loop PQBAP

4I1-I2=6-(ii)

∴ from equation (i) and (ii) we get

$$I_1 = 1.83A \quad I_2 = 1.95A$$

∴ the current in the 1Ω resistor is 0.13 A from Q to P

Q. 12 The force experienced by a current-carrying conductor placed in a magnetic eld is the largest when the angle between the conductor and the magnetic eld is :

Option 1:
60°

Option 2:
45°

Option 3:
90°

Option 4:
180°

Correct Answer:
90°

Solution:
The maximum force is exerted on a current-carrying conductor only when it is perpendicular to the direction of magnetic eld. No force acts on a current-carrying conductor when it is parallel to the magnetic eld.

Q. 13 Assume that 60% of the consumed energy is converted into light. Wavelength of sodium light = 590 nm. The number of photons emitted per second by a 10 W sodium vapour lamp is -

Option 1:
1.7×10^{20}

Option 2:
1.7×10^{19}

Option 3:
1.7×10^{-19}

Option 4:
1.7×10^{18}

Correct Answer:
1.7×10^{19}

Solution:

60% of $10W = 6W$ is converted into light.
Energy of single photon $= [6.63 \times 10^{-34} \times 3 \times 10^{8}] / [590 \times 10^{-9}]$
$= 3.371 \times 10^{-19} J$
Number of photons required to produce 6 W energy:
$n = 6/[3.371 \times 10^{-19}] = 1.7 \times 10^{19}$

Q. 14 An electron has a mass of $9.1 \times 10^{-31} kg$. It revolves around the nucleus in a circular orbit of radius 0.529×10^{-10} metre at a speed of $2.2 \times 10^{6} m/s$. The magnitude of its linear momentum in this motion is

Option 1:
$1.1 \times 10^{-34} kg - m/s$

Option 2:
$2.0 \times 10^{-24} kg - m/s$

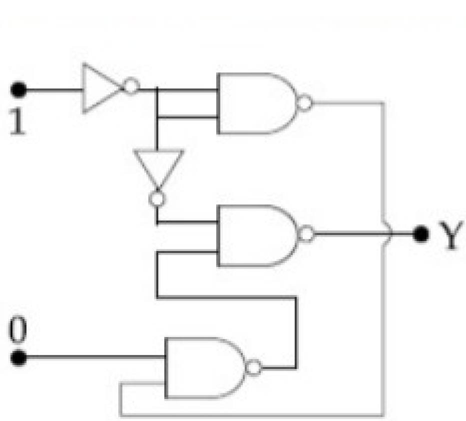
Option 3:
$4.0 \times 10^{-24} kg - m/s$

Option 4:
$4.0 \times 10^{-31} kg - m/s$

Correct Answer:
$2.0 \times 10^{-24} kg - m/s$

Solution:
Linear momentum $= mv = 9.1 \times 10^{-31} \times 2.2 \times 10^{6} = 2.0 \times 10^{-24} kg - m/s$

Q. 15 In the given circuit, value of Y is :



Option 1:
toggles between 0 and 1

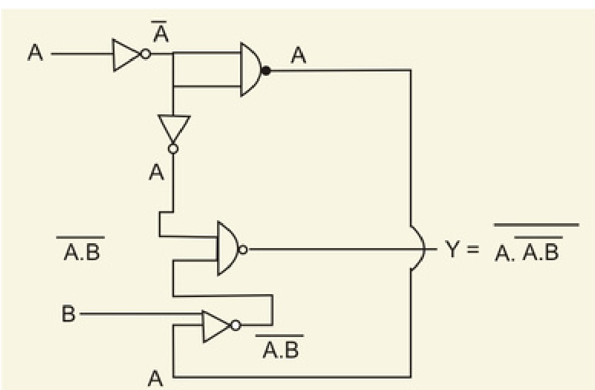
Option 2:
1

Option 3:
0

Option 4:
will not execute

Correct Answer:
0

Solution:



In NAND gate, when

$$y = \overline{A.\overline{A.B}}$$
$$Given$$
$$A = 1,\ \ B = 0$$
$$\Rightarrow A.B = 0$$
$$\overline{A.B} = 1\ and\ \ A.\overline{A.B} = 1$$
$$therefore\ \overline{A.\overline{A.B}} = 0$$

Hence the correct option is (3)

Chemistry

Q. 1 8g of oxygen has same number of atoms in:

Option 1:
3g O3

Option 2:
8g O3

Option 3:
16g O3

Option 4:
4g H2

Correct Answer:
8g O3

Solution:

8g O2 has $\frac{2 \times N_A \times 8}{32}$ atoms of O

$= \frac{N_A}{2}$ atoms of O

$$8\text{g of } O_3 \text{ has } \frac{3 \times N_A \times 8}{48} = \frac{N_A}{2} \text{ atoms of O}$$

Correct option is (2).

Q. 2 The number of nodal planes in a Px - orbital?

Option 1:
1

Option 2:
2

Option 3:
3

Option 4:
zero

Correct Answer:
1

Solution:
The angular and radial nodes of an orbital are l and (n- l - 1) respectively. For p- orbital l=1.

The correct option is (1).

Q. 3 A photon of 300 nm is absorbed by a gas which then re-emits two photons. One re- emitted photon has wavelength 496nm. Calculate energy of other photon re - emitted out .

Option 1:
$2.625 \times 10^{-19} J$

Option 2:
$759 \times 10^{-9} J$

Option 3:
2625 J

Option 4:
759 J

Correct Answer:
$2.625 \times 10^{-19} J$

Solution:
Total energy absorbed = total energy re-emitted out

$$\frac{hc}{300 \times 10^{-9}} = \frac{hc}{496 \times 10^{-9}} + \frac{hc}{\lambda_2}$$

$\lambda_2 = 759 \times 10^{-9}$m

Also, energy re-emitted in form of II photon

$$= \frac{hc}{\lambda} = \frac{6.626 \times 10^{34} \times 3 \times 10^8}{759 \times 10^{-9}} = 2.625 \times 10^{-19} J$$

The correct option is (1).

Q. 4 Ionisation enthaply is lowest for which of the following group of elements:

Option 1:
Alkali metals

Option 2:
Noble gas

Option 3:
Chalogens

Option 4:
Halogens

Correct Answer:
Alkali metals

Solution:
In periodic table, alkali metals have the lowest values of ionization enthalpy. This is because of the largest size of alkali metals. Because of this large size, the attraction between the nucleus the outremost electrons is less and thus removing an electron from the atom is easy. Thus ionization enthalpy is less for alkali metals.

The correct option is (1).

Q. 5 Which element posses non - spherical shells?

Option 1:
Li

Option 2:
Ba

Option 3:
B

Option 4:
He

Correct Answer:
B

Solution:
B has 1s22s22p1 con guration and p is non - spherical shell.

The correct option is (3).

Q. 6 Mass of calcium oxide required when it reacts with 852g of the $P4O_{10}$ is:

Option 1:
1050 g

Option 2:
105g

Option 3:
100 g

Option 4:
1008 g

Correct Answer:
1008 g

Solution:
$$6CaO + P_4O_{10} \rightarrow 2Ca_3(PO_4)_2$$

$\because$ 1 mole of $P4O_{10}$ requires 6 moles of CaO

$\therefore \frac{852}{284}$ moles of $P4O_{10}$ require $\frac{6 \times 852}{284}$ mole CaO

$$= \frac{6 \times 852}{284} \times 56 \text{ CaO}$$

$$= 1008 \text{ g CaO}$$

The correct option is (4).

Q. 7 0.5 g of fuming H2SO4 (Oleum) is diluted with water. This solution is completely neutralized by 26.7ml of 0.4N NaOH. The % of free SO3 in the sample is:

Option 1:
50%

Option 2:
10.6%

Option 3:
20.6%

Option 4:
40.6%

Correct Answer:
20.6%

Solution:
Meq. of H2SO4 + Meq. of SO3 = Meq. of NaOH

$$\frac{(0.5-a)}{49} \times 1000 + \frac{a}{\left(\frac{80}{2}\right)} \times 1000 = 26.7 \times 0.4$$

$\therefore a = 0.103$

$\therefore$ % of SO3 $= \dfrac{0.103}{0.5} \times 100$ = 20.6%

The correct option is (3).

Q. 8 The total number of protons, neutrons and electrons in 12g of ${}^{12}_{6}C$ is:

Option 1:
18

Option 2:
6.022×10^{23}

Option 3:
1.084×10^{25}

Option 4:
6.022×10^{22}

Correct Answer:
1.084×10^{25}

Solution:
${}^{12}_{6}C$ contains 6 NA protons, 6NA neutrons and 6NA electrons.

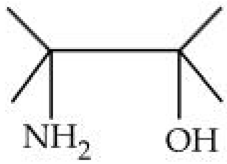
So, total number $= 6N_A + 6N_A + 6N_A$

$= 18N_A$

$$= 18 \times 6.022 \times 10^{23}$$

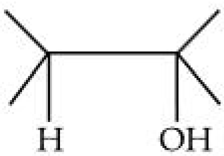
$$= 1.084 \times 10^{25}$$

Q. 9 The major product of the reaction



$\underrightarrow{NaNO_2/H_2SO_4}$ is

Option 1:



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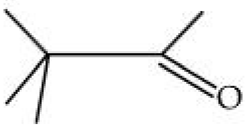

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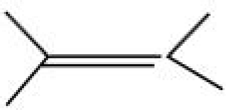
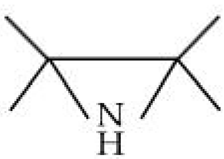
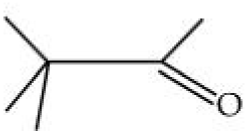


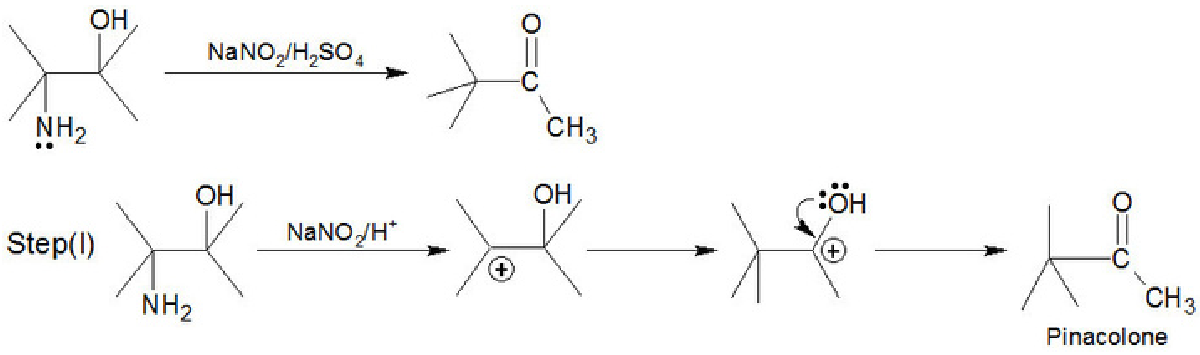
Option 4:



Correct Answer:



Solution:



Q. 10 An octahedral complex of Co^{3+} is diamagnetic. The hybridisation involved in the formation of the complex is :

Option 1:
sp3d2

Option 2:
dsp2

Option 3:
d2sp3

Option 4:
dsp3d

Correct Answer:
d2sp3

Solution: As we discussed in

Hybridisation - sp3d2 - square bipyramidal or octahedral d2sp3 - octahedral sp3 - tetradedral dsp2 - square planar - wherein sp3d2 - outer complex d2sp3 - inner complex sp3 - $[Ni(Cl)_4]^{2-}$

dsp2 - $[Pt(CN)_4]^{2-}$

diamagnetic octahedralcomplex

of Co^{3+} is d^2sp^3 hybridised.

Q. 11 At a certain temperature, only 50% HI is dissociated into H2 and I2 at equilibrium.The equilibrium constant is :

Option 1:
1.0

Option 2:
3.0

Option 3:
0.5

Option 4:
0.25

Correct Answer:
0.25

Solution:
For the reaction

$$2\mathrm{HI} \rightleftharpoons \mathrm{H}_2 + \mathrm{I}_2$$

Given that 50% of HI is dissociated i.e

$$2\mathrm{HI} \rightleftharpoons \mathrm{H}_2 + \mathrm{I}_2$$

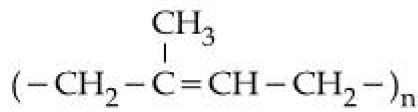
1-0.5 0.25 0.25

$$\mathrm{K_C} = \frac{0.25 \times 0.25}{0.5 \times 0.5} = 0.25$$

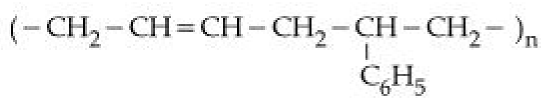
Hence, **the correct answer is Option (4)**

Q. 12 Structure of some important polymers are given. Which one represents Buna-S ?

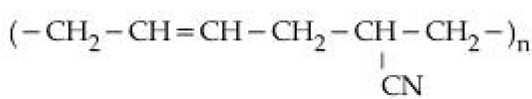
Option 1:



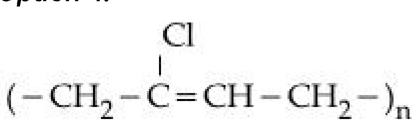
Option 2:



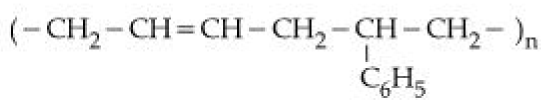
Option 3:



Option 4:



Correct Answer:



Solution:
As discussed in

Buna- N -

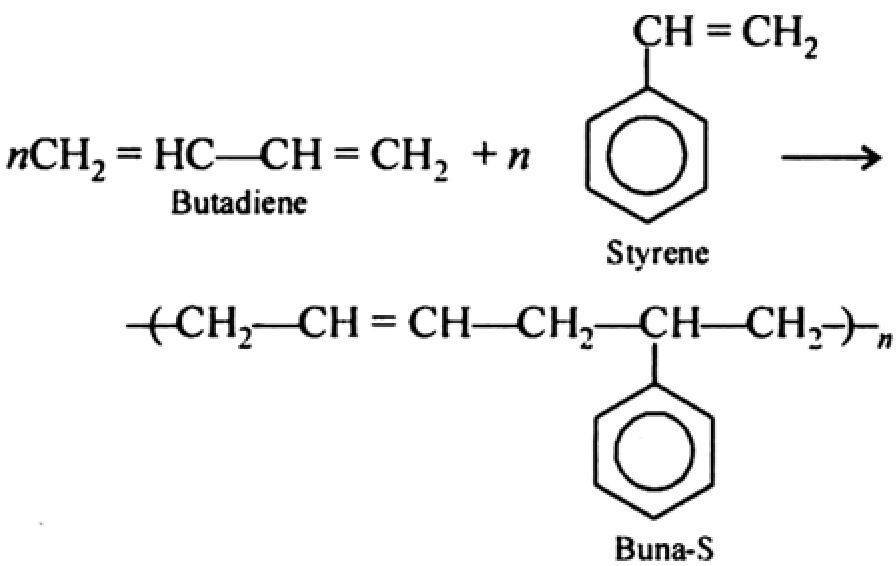
- Obtained by copolymerisation of 1,3- butadiene and acrylonitrile in the presence of peroxide.

- wherein

- Resistant to petrol, lubricating oil and organic solvents.

- Used in making oil seals, tank lining.

Solution



Therefore, The option (2) is correct.

Q. 13 Chloro compound of Vanadium has only spin magnetic moment of 1.73 BM. This Vanadium chloride has the formula :

(at. no. of V=23)

Option 1:
VCl2

Option 2:
VCl4

Option 3:
VCl3

Option 4:
VCl5

Correct Answer:
VCl4

Solution:
As discussed in

Magnetic Quantum Number (m) -

It gives informa on about the spa al orienta on of the orbital with respect to standard set of co-ordinate axis.

-

The value of magnetic moment is 1.73 BM

$1.73 = \sqrt{n\,(n+2)}$ where n is the number of unpaired electrons

$3 = n\,(n+2)$

After calculation n = 1

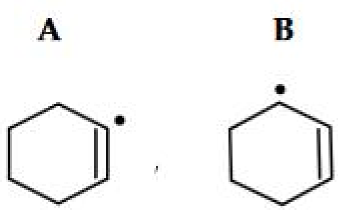
$V_{23} => 1S^2, 2S^2, 2p^6, 3S^2, 3p^6, 4S^2, 3d^3$

To obtain one unpaired electron V should be tetrapositive ion and the formula of its chlorid should be VCl_4

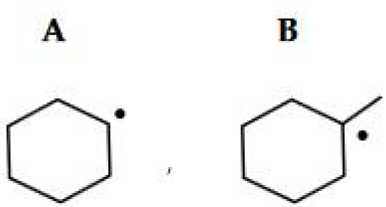
Option 2 is correct

Q. 14 In which of the following pairs A is more stable than B?

Option 1:

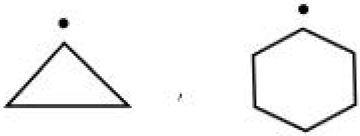


Option 2:



Option 3:

A B



Option 4:

A B

$Ph_3C^\bullet$, $(CH_3)_3C^\bullet$

Correct Answer:

A B

$Ph_3C^\bullet$, $(CH_3)_3C^\bullet$

Solution:
As we learnt in

Stability of alkyl free radical due to resonance -

More the no of resonating structure more is the stability.

-

Stability due to resonance -

More the no of resonating structure more is the stability.

-

$Ph_3C^\bullet$ is resonance stabilised while $(CH_3)_3C^\bullet$ is stable due to hyperconjugation. Since resonance is dominating over hyperconjugation $Ph_3C^\bullet$ is more stable than $(CH_3)_3C^\bullet$.

option(1):B is more stable than A as B involves resonance.

Option(2):B is more stable than A as B has more alpha hydrogen(7 to 4) which implies more number of hyperconjugation structures. This makes B more stable.
Option(3):B is more stable than A as a six-membered ring is more stable than a 3 membered ring due to lesser ring strain of six memebered ring.
Option(4):A is more stable than B due to possibility of resonance with each of the phenyl groups attached to C.B doesn't show any resonance.
Therefore, option (4) is correct.

Q. 15 For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by:

Option 1:
$(k_1 - k_2)\,[A]$

Option 2:
$k_1 k_2\,[A]$

Option 3:
$(k_1 + k_2)\,[A]$

Option 4:
$\left(\frac{k_1}{k_2}\right)[A]$

Correct Answer:
$\left(\frac{k_1}{k_2}\right)[A]$

Solution:

Rates in presence of stoichiometry of reactants/products -

When stoichiometry coe cients of reactants/ products are not equal to one, the rate of disappearance of & the rate of appearance of products is divided by their respective stoichiometric coe cients
- wherein

$$e.g.\ 2HI(g) \rightarrow H_2(g) + I_2(g)$$

$$r = \frac{-1}{2}.\frac{d}{dt}[HI]$$

$$= \frac{+d}{dt}[H2] = \frac{+d}{dt}[I_2]$$

Rate of Law = Dependence of Rate on Concentration -

The representation of rate of a reaction in terms of concentration of the reactants is known as Rate Law

or

The Rate Law is the expression in which reaction rate is given in terms of molar concentration of reactants with each term raised to some power, which may/maynot be equal to stoichiometric of the reacting species in a balanced chemical equation - wherein Formula:

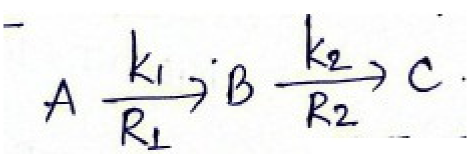
$$aA + bB \rightarrow cC + dD$$

$$Rate = \frac{dR}{dT}$$

$$= \alpha[A]^x.[B]^y$$

$$= k[A]^x.[B]^y$$

K= rate constant



$\frac{d[B]}{dt} = 0$ (given) [B]=?

$R_1 = k_1\,[A] \qquad R_2 = k_2\,[B]$

net rate of formation of $B = \frac{d[B]}{dt} = R_1 - R_2$

$$k_1[A] - k_2[B] = 0$$

$$k_1[A] = k_2[B]$$

$$[B] = \left(\frac{k_1}{k_2}\right)[A]$$

Maths

Q. 1 The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :

Option 1:
$\frac{1}{2}$

Option 2:
$\frac{1}{2\sqrt{2}}$

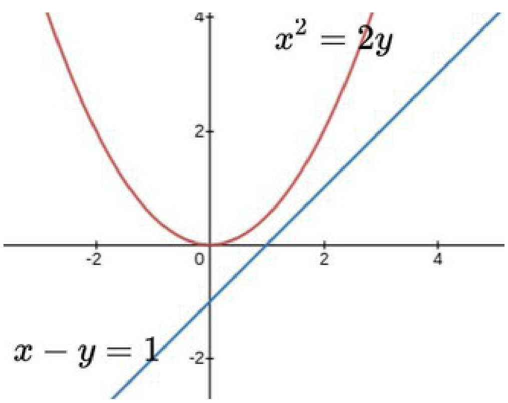
Option 3:
$\frac{1}{\sqrt{2}}$

Option 4:
0

Correct Answer:
$\frac{1}{2\sqrt{2}}$

Solution:
The shortest distance between curves is always along common normal.

the slope of the line



$$\frac{dy}{dx} = 1 = \text{slope of the line}$$

P is any point on the parabola, and also tangent pass through point P

slope of the tangent to the parabola

$$2x = 2\frac{dy}{dx}$$
$$\frac{dy}{dx} = x = 1$$
$$\Rightarrow y = \frac{1}{2}$$

$$\text{Point P} = \left(1, \frac{1}{2}\right)$$

$$\therefore \text{ Shortest distance } = \left|\frac{1 - \frac{1}{2} - 1}{\sqrt{1^2 + 1^2}}\right| = \frac{1}{2\sqrt{2}}$$

Q. 2 If the tangent to the curve, $y = f(x) = x \ \log_e x, (x > 0)$ at a point $(c, f(c))$ is parallel to the line - segment joining the points $(1, 0)$ and (e, e), then c is equal to :

Option 1:
$$\frac{e-1}{e}$$

Option 2:
$e^{\left(\frac{1}{e-1}\right)}$

Option 3:
$e^{\left(\frac{1}{1-e}\right)}$

Option 4:
$$\frac{1}{e-1}$$

Correct Answer:
$e^{\left(\frac{1}{e-1}\right)}$

Solution:

$$f(x) = x \log_e x$$

$$f'(x)|_{(c,f(c))} = \frac{e-0}{e-1}$$

$$f'(x) = 1 + \log_e x$$

$$f'(x)|_{(c,f(c))} = 1 + \log_e c = \frac{e}{e-1}$$

$$\log_e c = \frac{e-(e-1)}{e-1} = \frac{1}{e-1} \Rightarrow c = e^{\frac{1}{e-1}}$$

Q. 3 What is the value of $\lim_{x \to 0} \frac{\cos ax - \cos bx}{x^2} =$

Option 1:
$\frac{a^2 - b^2}{2}$

Option 2:
$\frac{b^2 - a^2}{2}$

Option 3:
$a^2 - b^2$

Option 4:
$b^2 - a^2$

Correct Answer:
$\frac{b^2 - a^2}{2}$

Solution:

Limit Using Expansion (Part 2) -

Limit Using Expansion (Part 2)

(vi) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots.$

(vii) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots.$

(viii) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \ldots\ldots.$

(ix) $\sin^{-1} x = x + \frac{1^2}{3!} \cdot x^3 + \frac{1^2 \cdot 3^2}{5!} \cdot x^5 + \frac{1^2 \cdot 3^2 \cdot 5^2}{7!} \cdot x^7 \ldots\ldots\ldots$

(x) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\ldots$

-

As we know

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\cdots$$

$$\lim_{x\to 0} \frac{\cos ax - \cos bx}{x^2} = \lim_{x\to 0} \frac{[1 - \frac{(ax)^2}{2!} + \frac{(ax)^4}{4!} - \cdots] - [1 - \frac{(bx)^2}{2!} + \frac{(bx)^4}{4!} - \cdots]}{x^2}$$

$$= \frac{b^2 - a^2}{2}$$

Q. 4 Let $x = (\theta + \cos\theta), y = (\theta - \sin\theta)$ then $\frac{d^2y}{dx^2}$ equals ?

Option 1:
$\frac{\sin\theta + \cos\theta + 1}{(1 - \sin\theta)^3}$

Option 2:
$\frac{\sin\theta - \cos\theta + 1}{(1 - \sin\theta)^3}$

Option 3:
$\frac{\sin\theta - \cos\theta - 1}{(1 - \sin\theta)^3}$

Option 4:
$$\frac{\sin\theta+\cos\theta-1}{(1-\sin\theta)^3}$$

Correct Answer:
$$\frac{\sin\theta+\cos\theta-1}{(1-\sin\theta)^3}$$

Solution:
As we have learnt,

Second order derivative for parametric function -

When we nd

$$\frac{dy}{dx}=F(t)\ then\ \frac{d^2y}{dx^2}=\frac{\frac{d}{dt}F(t)}{\frac{dx}{dt}}$$

-

$$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\frac{d}{d\theta}(\theta-\sin\theta)}{\frac{d}{d\theta}(\theta+\cos\theta)}=\frac{1-\cos\theta}{1-\sin\theta}$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1-\cos\theta}{1-\sin\theta}\right)=\frac{\frac{d}{d\theta}\left(\frac{1-\cos\theta}{1-\sin\theta}\right)}{\frac{dx}{d\theta}}$$

$$\frac{d^2y}{dx^2}=\frac{(1-\sin\theta)(\sin\theta)-(1-\cos\theta)(-\cos\theta)}{(1-\sin\theta)^3}=\frac{\sin\theta+\cos\theta-1}{(1-\sin\theta)^3}$$

Q. 5 The equation of the curve passing through the origin and satisfying the di erential equation $(1+x^2)\frac{dy}{dx}+2xy = 4x^2$ is :

Option 1:
$(1+x^2)y=x^3$

Option 2:
$3(1+x^2)y = 2x^3$

Option 3:
$(1+x^2)y = 3x^3$

Option 4:
$3(1+x^2)y = 4x^3$

Correct Answer:
$3(1+x^2)y = 4x^3$

Solution:
Given DE is

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = 4x^2$$
$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$$

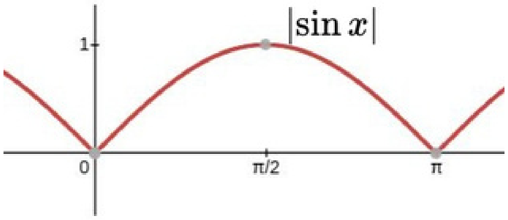
This DE is in the form of linear DE

$$\text{I.F} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log\left(1+x^2\right)} = 1+x^2$$
$$y\left(1+x^2\right) = \int \frac{4x^2}{1+x^2} \times 1 + x^2 + C$$
$$\Rightarrow y\left(1+x^2\right) = \frac{4x^3}{3} + C$$
$$\Rightarrow \text{ Required curve is}$$
$$3y\left(1+x^2\right) = 4x^3 (\because \text{C} = 0)$$

Q. 6 The value of the integral $\int_0^{\pi} |\sin 2x|\, dx$ is ______.

Correct Answer:
2

Solution:
$$\text{put } 2\text{x} = \text{t} \Rightarrow 2dx = dt$$
$$I = \int_0^{2\pi} |\frac{\sin t}{2}|dt$$
$$I = 2 \times \frac{1}{2}\int_0^{\pi} |\sin t|dt$$



$$I=\int_0^{\pi}\sin t dt=-\cos t|_0^{\pi}$$
$$I=-\cos\pi-(-\cos 0)=2$$

Q. 7 $\int \frac{dx}{\cos x+\sqrt{3}\sin x}\ equals$

Option 1:
$\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

Option 2:
$\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

Option 3:
$\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

Option 4:
$\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+C$

Correct Answer:
$\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$

Solution:
As the learnt in concept

Integrals for Trigonometric functions -

$$\frac{\mathrm{d} }{\mathrm{d} x}\left (-cosx \right)= sinx$$

$$\therefore \int sinx dx=-cosx+c$$

-

$$I=\int\frac{dx}{cosx+\sqrt{3}sinx}dx$$

$$cosx+\sqrt{3}sinx=2cos\left(x-\frac{\pi}{3}\right)$$

$$I=\frac{1}{2}\int\frac{dx}{\left(cosx-\frac{\pi}{3}\right)}dx$$

$$I=\frac{1}{2}\int sec\left(x-\frac{\pi}{3}\right)dx$$

$$I=\frac{1}{2}log\,tan\left(\frac{x}{2}-\frac{\pi}{6}+\frac{\pi}{4}\right)+C$$

$$I=\frac{1}{2}log\,tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+C$$

Q. 8 If the area enclosed between the curves $y=kx^2$ and $x=ky^2$,(k>0), is 1 sq. unit. Then k is :

Option 1:

$\frac{\sqrt{3}}{2}$

Option 2:

$\frac{1}{\sqrt{3}}$

Option 3:

$\sqrt{3}$

Option 4:

$\frac{2}{\sqrt{3}}$

Correct Answer:

$\frac{1}{\sqrt{3}}$

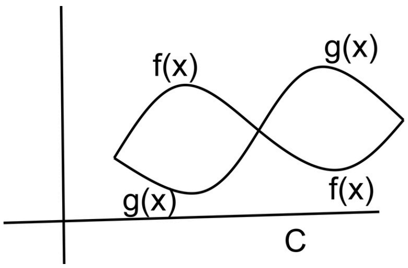
Solution:

Area between two curves -

$$If\ f(x) \geqslant g(x)$$
$$in[a, c)\ and\ g(x) \geqslant f(x)\ in(c, b]$$
$$Then\ area =$$

$$\int_a^c (f(x) - g(x))\,dx + \int_c^b (g(x) - f(x))\,dx$$

- wherein



Area bounded by $y^2 = 4ax$ and $x^2 = 4by$ is $\left|\frac{16ab}{3}\right|$ (standard formula / result)

put $4a = \frac{1}{k}$ and $4b = \frac{1}{k}$

Area $= \left|\frac{16 \cdot \frac{1}{4k} \cdot \frac{1}{4k}}{3}\right| = 1$

$$k = \frac{1}{\sqrt{3}}$$

Q. 9 Let L be a tangent line to the parabola $y^2 = 4x - 20$ at $(6, 2)$. If L is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1,$ then the value of b is equal to :

Option 1:
20

Option 2:
14

Option 3:
11

Option 4:
16

Correct Answer:
14

Solution:
Tangent to the parabola

2y = 2(x + 6) - 20

y = x - 4

Condition for tangent to the ellipse

16 = 2(1)2 + b

b = 14

Q. 10 The image of the point (3,5) in the line $x - y + 1 = 0$, lies on :

Option 1:
$(x-2)^2 + (y-4)^2 = 4$

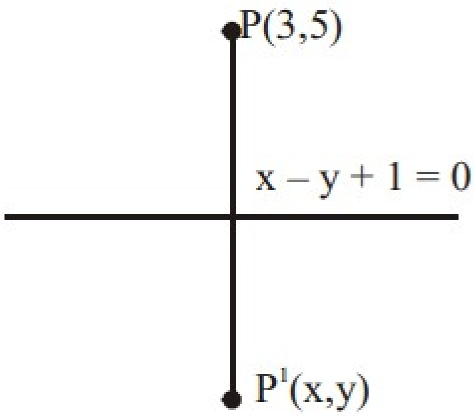
Option 2:
$(x-2)^2 + (y-2)^2 = 12$

Option 3:
$(x-4)^2 + (y+2)^2 = 16$

Option 4:
$(x-4)^2 + (y-4)^2 = 8$

Correct Answer:
$(x-2)^2 + (y-4)^2 = 4$

Solution:



$$\frac{x-3}{1}=\frac{y-5}{-1}=-2\left(\frac{3-5+1}{1+1}\right)$$
So, $x=4, y=4$

Hence, $(x-2)^2+(y-4)^2=4$

Q. 11 If the circle $x^2+y^2-6x-8y+\left(25-a^2\right)=0$ touches the axis of x, then a equals.

Option 1:
0

Option 2:
± 4

Option 3:
± 2

Option 4:
\pm 3

Correct Answer:
± 4

Solution:
$x^2+y^2-6x-8y+\left(25-a^2\right)=0$
Radius $=4=\sqrt{9+16+\left(25-a^2\right)}$
$\Rightarrow a=\pm 4$

Q. 12 In an ellipse, the distance between its focii is 6 and minor axis is 8. Then its eccentricity is,

Option 1:
3/5

Option 2:
1/2

Option 3:
4/5

Option 4:
1/$\sqrt{5}$

Correct Answer:
3/5

Solution:
As we learnt in

Coordinates of foci -

$\pm ae, o$

- wherein

For the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Length of major

axis -

$2b$

- wherein

$B\rightarrow$ Semi minor axis

Eccentricity -

$e=\sqrt{1-\frac{b^2}{a^2}}$

- wherein

For the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Distance between its focii is 6.

2ae = 6; ae = 3

minor axis, 2b = 8; b=4

b2 = a2(1-e2)

b2 = a2- a2e2

a2=25$\longrightarrow$ a=5

$$e=\sqrt{1-\frac{b^2}{a^2}}=\frac{3}{5}$$

Q. 13 The perpendicular distance from the origin to the plane containing the two lines, $\frac{x+2}{3}=\frac{y-2}{5}=\frac{z+5}{7}$ and $\frac{x-1}{1}=\frac{y-4}{4}=\frac{z+4}{7}$ is :

Option 1:
$\frac{11}{\sqrt{6}}$

Option 2:
$6\sqrt{11}$

Option 3:
11

Option 4:
$11\sqrt{6}$

Correct Answer:
$\frac{11}{\sqrt{6}}$

Solution:

Condition for line to be lie in plane -

$\vec{b} \cdot \vec{n} = 0$ and $\vec{a} \cdot \vec{n} = d$ or

$a_1 a + b_1 b + c_1 c = 0$ and $a_1 x_1 + b_1 y_1 + c_1 z_1 + d = 0$

-

$L_1 \ pass \ through \ (-2, 2, -5) \ and \ L_2 \ pass \ through \ (1, 4, -4)$
$Direction ratio of \, L_1 i.e. \, D_1 = (3, 5, 7) \, and \, Direction ratio of \, L_2, \, D_2 = (1, 4, 7)$
$Equation \ of \ plane \ will \ be.$

$$P = \begin{vmatrix} x - (-2) & y - 2 & z - (-5) \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0$$

$P = x - 2y + z + 11 = 0$
$Now, \ perpendicular \ distance \ from \, the \ origin \ (0, 0, 0)$

$$\frac{|11|}{\sqrt{(1)^2 + (-2)^2 + (1)^2}} = \frac{11}{\sqrt{6}}$$

Q. 14 The lines $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

Option 1:
do not intersect for any values of l and m

Option 2:
intersect for all values of l and m

Option 3:
intersect for the values when l=2 and $m = \frac{1}{2}$

Option 4:
intersect for the values when l=2 and m=2

Correct Answer:
do not intersect for any values of l and m

Solution:
$\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$

$= (2l + 1)\hat{i} - \hat{j} + l\,\hat{k}$

$\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$

$= (m+2)\hat{i} + (m-1)\hat{j} - m\hat{k}$

For intersection

$$\begin{aligned} &1 + 2\ell = 2 + \text{m} && \ldots(i) \\ &-1 = m - 1 && \ldots(ii) \\ &\ell = -\text{m} && \ldots(iii) \\ &\text{from (ii) } m = 0 \end{aligned}$$

$$\text{from (iii) } \ell = 0$$

These values of m and l do not satisfy equation (1). Hence the two lines do not intersect for any values of l and m

---

Q. 15 If $|\vec{a}| = 5, \left|\vec{b}\right| = 4, |\vec{c}| = 3$ thus what will be the value of $|\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}|,$ given that $\vec{a} + \vec{b} + \vec{c} = 0$

Option 1:
25

Option 2:
50

Option 3:
–25

Option 4:
–50

Correct Answer:
25

Solution:
As we have learned

Properties of Scalar Product -

$\vec{a}.\vec{a} = |a|^2 = \vec{a}^2 \;\; Commutative\ Property$

$\vec{a}.\vec{b} = \vec{b}.\vec{a} \;\; Distributive\ Property$

-

$$(\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=0$$
$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$$

$$\Rightarrow 25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$$

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-25$$

$$|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|=25$$

Q. 16 The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is :

Option 1:
16.8

Option 2:
16.0

Option 3:
15.8

Option 4:
14.0

Correct Answer:
14.0

Solution:
As we learnt in

ARITHMETIC Mean -

For the values x1, x2, ...xn of the variant x the arithmetic mean is given by

$$\bar{x}=\frac{x_1+x_2+x_3+\cdots+x_n}{n}$$

in case of discrete data.

-

$$\frac{\sum xi}{16}=16$$

$$\Rightarrow\sum_{i=1}^{18} v_i=(16\times 16-16)+(3+4+5)=252$$

number of observations=18

$$mean=\frac{252}{18}=14$$

Q. 17 A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is :

Option 1:

$\frac{9}{16}$

Option 2:

$\frac{7}{16}$

Option 3:

$\frac{9}{32}$

Option 4:

$\frac{7}{8}$

Correct Answer:

$\frac{7}{16}$

Solution:

As we learned

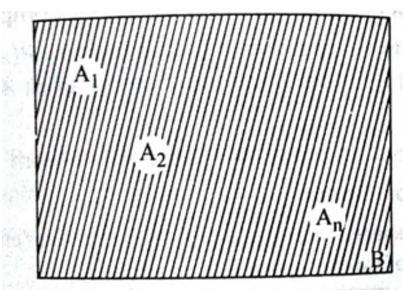
BAYE'S Theorem -

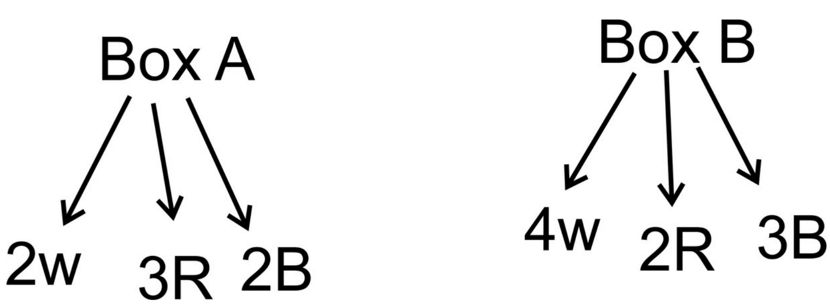
If E1, E2, E3……En be n mutually exclusive and exhaustive events and A is an event which occurs together with either E1, E2, E3……En from a portion of the sample space S and A be any event then

$$P\left(\frac{E_k}{A}\right)=\frac{P\left(E_k\right)\cdot P\left(\frac{A}{E_k}\right)}{P\left(E_1\right)\cdot P\left(\frac{A}{E_1}\right)+P\left(E_2\right)\cdot P\left(\frac{A}{E_2}\right)+\cdots P\left(E_k\right)\cdot P\left(\frac{A}{E_k}\right)}$$

where S = $E_1\cup E_2\cup\cdots E_k$ and S is sample space.

- wherein





Using Baye's Theorem

the probability of drawing a white ball and then a red ball from bag B is given by

$$=\frac{{}^4C_1\times{}^2C_1}{{}^9C_2}=\frac{2}{9}$$

the probability of drawing a white ball and then a red ball from bag A is given by

$$=\frac{{}^2C_1\times{}^3C_1}{{}^7C_2}=\frac{2}{7}$$

Hence, the probability of drawing a white ball and then a red ball from bag B is given by

$$= \frac{\frac{2}{9}}{\frac{2}{7} + \frac{2}{9}} = \frac{7}{16}$$

Q. 18 If the probability of hitting a target by a shooter, in any shot, is $\frac{1}{3}$, then the minimum number of independent shots at the target required by him so that the probability of hitting the target at least once is greater that $\frac{5}{6}$, is:

Option 1:
6

Option 2:
5

Option 3:
4

Option 4:
3

Correct Answer:
5

Solution:

Binomial Theorem on Probability -

If an experiment is repeated n times under similar conditions we say that n trials of the experiment have been made.
Let E be an event.

P= the Probability of occurrence of event E in one trial.

q = 1- p = probability of non occurrence of event E in one trial such that p+q = 1

x= number of successes.

-

Binomial Theorem on Probability -

Then

$P(X=r)$ or $P(r)$

$= {}^{n}C_{\Upsilon} \cdot P^{r} \cdot q^{n-r}$

-

From the concept

$1 -_0^n C(\frac{1}{3})^0(\frac{2}{3})^n > \frac{5}{6}$

$\frac{1}{6} > (\frac{2}{3})^n$

$=> 0.16 > (\frac{2}{3})^n$

$n_{min} = 5$

Q. 19 The statement $\sim(p \leftrightarrow \sim q)$ is :

Option 1:
a tautology

Option 2:
a fallacy

Option 3:
equivalent to $p \leftrightarrow q$

Option 4:
equivalent to $\sim p \leftrightarrow q$

Correct Answer:
equivalent to $p \leftrightarrow q$

Solution:
As we learnt in

Truth Table of "if and only if" -

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

-

Truth Table of "NOT" operator -

p	T
T	T
F	T

-

P	q	$\sim q$	$(p \leftrightarrow \sim q)$	$\sim (p \leftrightarrow \sim q)$
T	T	f	f	T
T	f	T	T	f
f	T	f	T	f
f	f	T	f	T

This is the same truth table as $p \leftrightarrow q$

Q. 20 The value of $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$,

$|x|<\frac{1}{2}, x \neq 0$ is equal to:

Option 1:

$\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

Option 2:
$\frac{\pi}{4}+\cos^{-1}x^2$

Option 3:
$\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

Option 4:
$\frac{\pi}{4}-\cos^{-1}x^2$

Correct Answer:
$\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$

Solution:
As we learnt in

Trigonometric Ratios of Submultiples of an Angle -

$$\cos A=\cos^2\tfrac{1}{2}A-\sin^2\tfrac{1}{2}A$$
$$\sin A=2\sin\tfrac{1}{2}A\cos\tfrac{1}{2}A$$
$$\tan A=\frac{2\tan\frac{1}{2}A}{1-\tan^2\frac{1}{2}A}$$
$$\cos A=2\cos^2\tfrac{1}{2}A-1$$
$$\cos A=1-\sin^2\tfrac{1}{2}A$$
$$2\sin^2\tfrac{1}{2}A=1-\cos A$$
$$2\cos^2\tfrac{1}{2}A=1+\cos A$$

- wherein

This shows the formulae for half angles and their doubles.

$$\tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$$

Let $x^2=\cos 2t$

As, $x^2>0,\quad 2t\varepsilon(0.\frac{\pi}{2})\ \Rightarrow t\varepsilon(0.\frac{\pi}{4})$ ....(i)

Then, the expression becomes $= \tan^{-1}\left(\frac{\sqrt{1+\cos 2t}\ +\sqrt{1-\cos 2t}}{\sqrt{1+\cos 2t}\ -\sqrt{1-\cos 2t}}\right)$

$$= \tan^{-1}\frac{\sqrt{2\cos^2 t}\ +\sqrt{2\sin^2 t}}{\sqrt{2\cos^2 t}\ -\sqrt{2\sin^2 t}}$$

$$= \tan^{-1}\frac{|\cos t|\ \ + |\sin t|}{|\cos t|\ \ - |\sin t|}$$

$$= \tan^{-1}\frac{1+|\tan t|}{1-|\tan t|}$$ - {Dividing numerator and denominator by |cos t|}

$$= \tan^{-1}\frac{1+\tan t}{1-\tan t}$$ - {From (i)}

$$= \tan^{-1}\tan(\frac{\pi}{4}+\frac{1}{2}\ \cos^{-1}\ x^2)$$

$$= \frac{\pi}{4}+\frac{1}{2}\ \cos^{-1}\ x^2$$

Q. 21 An aeroplane ying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60° from a point on the ground. If, after ve seconds, its elevation from the same point, is 30°, then the speed (in m/s) of the aeroplane, is :

Option 1:
1500

Option 2:
400

Option 3:
750

Option 4:
720

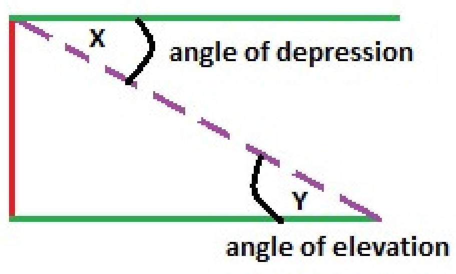
Correct Answer:
400

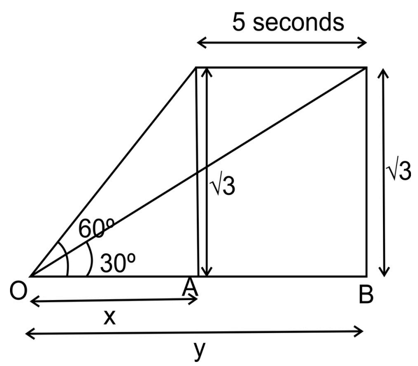
Solution:

Angle of Elevation -

If an object is above the horizontal line from the eye, we have to raise our head to view the object.

- wherein





$$\tan 60^{\circ} = \frac{\sqrt{3}}{x} = \sqrt{3}$$

$$\Rightarrow x = 1$$

$$\tan 30^{\circ} = \frac{\sqrt{3}}{y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow y = 3$$

$$AB = 2\ kms \quad also \quad Time = 5\ \ seconds$$

$$speed = \frac{distance}{time} = \frac{2 \times 1000}{5} = 400\ m/s$$

Q. 22 The number of real values of λ for which the system of linear equations

2x+4y−λz=0 4x+λy+2z=0 λx+2y+2z=0 has in nitely many solutions, is :

Correct Answer: 1

Solution:
As we learnt in

By using the concept of

Cramer's rule for solving system of linear equations -

When $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$,

then the system of equations has in nite solutions.

- wherein

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

and

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$\Delta_1, \Delta_2, \Delta_3$ are obtained by replacing column 1,2,3 of Δ by (d_1, d_2, d_3) column

$$\begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2\begin{vmatrix} \lambda & 2 \\ 2 & 2 \end{vmatrix} - 4\begin{vmatrix} 4 & 2 \\ \lambda & 2 \end{vmatrix} - \lambda\begin{vmatrix} 4 & \lambda \\ \lambda & 2 \end{vmatrix} = 0$$

$\Rightarrow 2(2\lambda - 4) - 4(8 - 2\lambda) - \lambda(8 - \lambda^2) = 0$

$\Rightarrow 4\lambda - 8 - 32 + 8\lambda - 8\lambda + \lambda^3 = 0$

$\Rightarrow \lambda^3 + 4\lambda - 40 = 0$

It will give only one real value of λ

Q. 23

If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to :

Option 1:

$\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Option 2:

$\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

Option 3:

$\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

Option 4:

$\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Correct Answer:

$\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Solution:

As we leant in

Multiplication of matrices -

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} \begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{pmatrix} =$$

$$\begin{pmatrix} a11.b11+a12.b21+a13.b31 & a11.b12+a12.b22+a13.b32 & a11.b13+a12.b23+a13.b33 \\ a21.b11+a22.b21+a23.b31 & a21.b12+a22.b22+a23.b32 & a21.b13+a22.b23+a23.b33 \\ a31.b11+a32.b21+a33.b31 & a31.b12+a32.b22+a33.b32 & a31.b13+a32.b23+a33.b33 \end{pmatrix}$$

-

Adjoint of a square matrix -

Transpose of the matrix of co-factors of elements of A is called the adjoint of A

- wherein

$$\mathrm{adj}(\mathbf{A}) = \mathbf{C}^{\mathsf{T}} = \begin{pmatrix} +\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ +\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & +\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & -4 \\ -4 & 1 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

adj A= Transpose of cofactors

so that $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Q. 24 If $f(x) = \dfrac{5x-3}{2x+1}$; Find the inverse of f(x)

Option 1:
$$\frac{x+3}{2x-5}; x \neq 5/2$$

Option 2:
$$\frac{-(x+3)}{2x-5}; x \neq 5/2$$

Option 3:
$$\frac{(x+3)}{2x-5}; x \neq 5/2$$

Option 4:
$$\frac{(x-3)}{2x-5}; x \neq 5/2$$

Correct Answer:
$$\frac{-(x+3)}{2x-5}; x \neq 5/2$$

Solution:
As we have learned

Property of Inverse -

The inverse of a bijection is also a bijection.

-

$$y = \frac{5x-3}{2x+1}$$

$$y - 2.5 = \frac{5x-3}{2x+1} - 2.5$$

$$y - \frac{5}{2} = \frac{5x - 3 - 5x - 2.5}{2x+1}$$

$$\frac{2y-5}{2} = \frac{-5.5}{2x+1}$$

$$2x + 1 = \frac{-11}{2y-5}$$

$$2x = \frac{-11}{2y-5} - 1$$

$$2x = \frac{-11 - 2y + 5}{2y-5}$$

$$\Rightarrow x = \frac{-3-y}{2y-5} = \frac{-(y+3)}{2y-5}$$

Now we can say inverse is equal to

$$y = \frac{-(x+3)}{2x-5}$$

Q. 25 Let ω be a complex number such that

$2w + 1 = z$ where $z = \sqrt{-3}$ if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k,$$

then k is equal to :

Option 1:
z

Option 2:
-1

Option 3:
1

Option 4:
$-z$

Correct Answer:
$-z$

Solution:
We have, $2\omega + 1 = \sqrt{3}i$

$$\Rightarrow \omega = \frac{\sqrt{3}i - 1}{2}$$

Using, Multiplication of Complex Numbers -

(a+ib)(c+id)=(ac-bd)+i(bc+ad)

-

$$\Rightarrow \omega^2 = \frac{-2 - 2\sqrt{3}i}{4} \Rightarrow \frac{-1 - \sqrt{3}i}{2}$$

$$\Rightarrow \omega^3 = -\frac{(\sqrt{3}i + 1)(\sqrt{3}i - 1)}{4} = 1$$

In order to nd out k, we have to nd the value of:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}$$

Here, $-\omega^2 - 1 = \frac{-1 + \sqrt{3}i}{2} = \omega$

So, $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

After expansion of determinant, we get,

$\omega^2 - \omega + \omega^2 - \omega + \omega^2 - \omega = 3(\omega^2 - \omega)$=$\omega^2 - \omega + \omega^2 - \omega + \omega^2 - \omega = 3(\omega^2 - \omega) = -3\sqrt{3}i$

Therefore, $k = -\sqrt{3}i = -z$

Q. 26 If equations

$ax^2+bx+c=0 (a, b, c \epsilon R, a \neq 0)$ and $2x^2+3x+4=0$

Have a common root ,then a:b:c equals :

Option 1:
$1:2:3$

Option 2:
$2:3:4$

Option 3:
$4:3:2$

Option 4:
$3:2:1$

Correct Answer:
$2:3:4$

Solution:
As we have learned

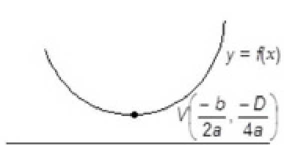
Quadratic Expression Graph when a > 0 & D < 0 -

No Real and Equal root of

$f\left(x\right)=ax^2+bx+c$

& $D=b^2-4ac$

- wherein



Condition for both roots common -

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

- wherein

$ax^2 + bx + c = 0$ &

$a'x^2 + b'x + c' = 0$

are the 2 equations

$2x^2 + 3x + 4 = 0$ has determinant = 9-32 = -23 < 0

So , non real roots which means both roots are common (as complex roots occurs in conjugate)

So, a:b:c = 2:3:4

Q. 27 The number of integers greater than 6,000 that can be formed, using the digits 3, 5, 6,7 and 8, without repetition, is :

Option 1:
216

Option 2:
192

Option 3:
120

Option 4:
72

Correct Answer:
192

Solution:

APPLICATION OF PERMUTATION-II -

-

Four-digit numbers can be

$\overline{}\ \overline{}\ \overline{}\ \overline{}$

$3 \times 4 \times 3 \times 2 = 72$

Five-digit numbers

$\overline{3} \times \overline{4} \times \overline{3} \times \overline{2} \times \overline{1} = 120$

Total = 192

The correct option is 2.

Q. 28 Let $(x+10)^{50}+(x-10)^{50}=a_0+a_1x+a_2x^2+\ldots\ldots\ldots\ldots+a_{50}x^{50}$, for all $x \epsilon R$; then $\frac{a_2}{a_0}$ is equal to :

Option 1:
12

Option 2:
12.75

Option 3:
12.25

Option 4:
12.5

Correct Answer:
12.25

Solution:

Expression of Binomial Theorem -

$$(x+a)^n = {}^nc_0x^na^0 + {}^nc_1x^{n-1}a^1 + {}^nc_2x^{n-2}a^2x - - - - - {}^nc_nx^0a^n$$

- wherein for n +ve integral . General

Term in the expansion of (x+a)^n -

$$T_{r+1} =^n c_r \cdot x^{n-r} \cdot a^r$$

- wherein

Where $r \geqslant 0 \, and \, r \leqslant n$

$$r = 0, 1, 2, - - - - n$$

Properties of Binomial Theorem -

$$(x+a)^n + (x-a)^n = 2\left({}^n c_0 \, x^n +^n c_2 \, x^{n-2} \, a^2 + - - -\right)$$

- wherein

Sum of odd terms or even Binomial coe cients

$$(10+x)^{50} + (10-x)^{50}$$

$$=> a_2 = 2 \cdot \ {}^{50}C_2 \, 10^{48}$$

$$=> a_0 = 2 \cdot 10^{50}$$

$$\frac{a_2}{a_0} = \frac{2 \cdot^{50} C_2 \, 10^{48}}{2 \cdot 10^{50}} = 12.25$$

Q. 29 Let

Sn $=\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}.......+\frac{1+2........+n}{1^3+2^3.......+n^3}$.

If 100 Sn=n, then n is equal to :

Correct Answer: 199

Solution:
As we learnt in

Summation of series of natural numbers -

$$\sum_{k=1}^{n} K=\frac{1}{2}n\left(n+1\right)$$

- wherein

Sum of rst n natural numbers

$$1+2+3+4+-------+n=\frac{n(n+1)}{2}$$

Summation of series of natural numbers -

$$\sum_{k=1}^{n} K^3=\frac{1}{4}n^2\left(n+1\right)^2$$

- wherein

Sum of cubes of rst n natural numbers

$$1^3+2^3+3^3+4^3+-------+n^3=\frac{n^2(n+1)^2}{4}$$

$$\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+---+\frac{1+2+3+--+n}{1^3+2^3+--+n^3}$$

Its T_n term is $\frac{1+2+3+--n}{1^3+2^3+3^3+--n^3}$

$$=\frac{\left(\frac{n(n+1)}{2}\right)}{\left[\frac{n(n+1)}{2}\right]^2}$$

$$=\frac{1}{\frac{n(n+1)}{2}}$$

$$=\frac{2}{n\,(n+1)}$$

$$T_n=2\left[\frac{n+1-n}{n\,(n+1)}\right]$$

$$=2\left[\frac{1}{n}-\frac{1}{n+1}\right]$$

$$T_1=2\left[\frac{1}{1}-\frac{1}{2}\right]$$

$$T_2=\left[\frac{1}{2}-\frac{1}{3}\right]$$

$$T_n=2\left[\frac{1}{n}-\frac{1}{n+1}\right]$$

$$S_n=(T_1+T_2+T_3---T_n)$$

$$=2\left[1-\frac{1}{n+1}\right]$$

$$=2\left(\frac{n+1-1}{n+1}\right)$$

$$=\left(\frac{2n}{n+1}\right)$$

Now $100S_n=\frac{2n}{n+1}\times 100=n(given)$

$\therefore\ n+1=200$

$n = 199$

Q. 30 If $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots\ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$,

then the sum of the rst 15 terms of this A.P. is :

Correct Answer: 200

Solution:

General term of an A.P. -

$T_n = a + (n-1)\,d$

- wherein

$a \longrightarrow$ First term

$n \longrightarrow$ number of term

$d \longrightarrow$ common di erence

Sum of n terms of an

AP -

$$S_n = \frac{n}{2}\left[2a + (n-1)\,d\right]$$

and

Sum of n terms of an AP

$$S_n = \frac{n}{2}\left[a + l\right]$$

- wherein

$a \longrightarrow$ rst term

$d \longrightarrow$ common di erence

$n \longrightarrow$ number of terms

Given ,

$$a_1 + a_7 + a_{16} = 40$$

$$=> a_1 + (a_1 + 6d) + (a_1 + 15d) = 40$$

$$=> a_1 + 7d = \frac{40}{3}$$..................(1)

We have to nd out ,

$a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots\ldots, a_{15}$

$$=> \frac{15}{2}[a_1 + a_{15}] = \frac{15}{2}[a_1 + a_1 + 14d] = 15(a_1 + 7d)$$..............(2)

Substituting the value of (1) in (2),

$$=> 15 \times \frac{40}{3} = 200$$