

## Physics

## SECTION - A

1. A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of  $5 \text{ ms}^{-1}$ . Neglecting the air resistance, the speed with which the stone hits the ground will be  $\text{ms}^{-1}$  (given,  $g = 10 \text{ ms}^{-2}$ ).

(1) 15

(2) 20

(3) 30

(4) 25

Sol. (1)

Along vertical direction

$$u_y = 0$$

$$v_y^2 = u_y^2 + 2a_y g_y$$

$$a_y = +g$$

$$= (0)^2 + 2 \times 10 \times 10$$

$$v_y = ?$$

$$v_y^2 = 200$$

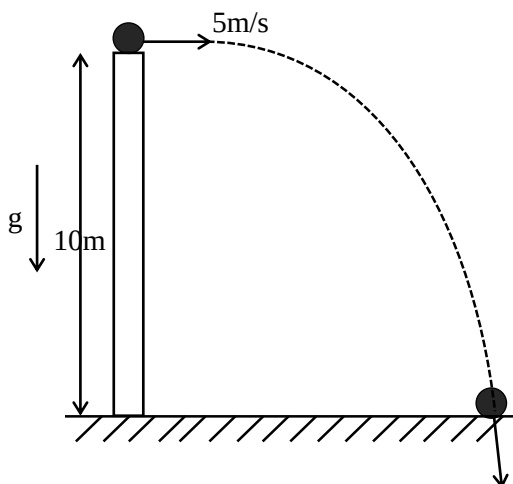
$$s_y = 10 \text{ m}$$

$$v_y^2 = 200$$

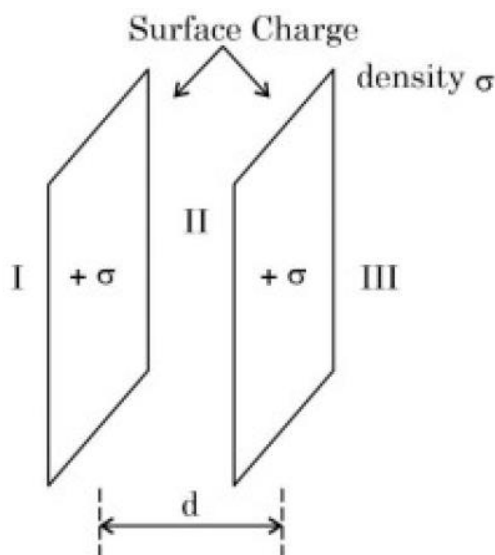
$$\therefore v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{25 + 200} = \sqrt{225}$$

$$= 15 \text{ m/s}$$



2. Let  $\sigma$  be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different region  $E_I$ ,  $E_{II}$  and  $E_{III}$  are:



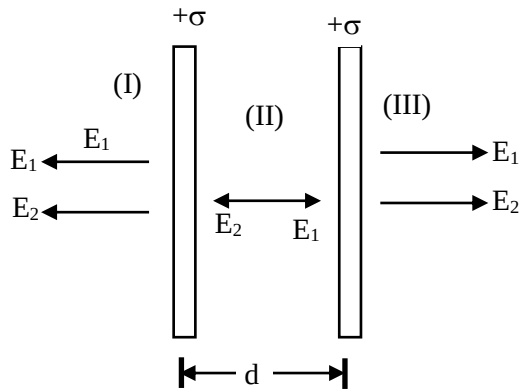
$$(1) \vec{E}_I = \frac{2\sigma}{\epsilon_0} \hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{2\sigma}{\epsilon_0} \hat{n}$$

$$(2) \vec{E}_I = \frac{\sigma}{2\epsilon_0} \hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{2\epsilon_0} \hat{n}$$

$$(3) \vec{E}_I = -\frac{\sigma}{\epsilon_0} \hat{n}, \vec{E}_{II} = 0, \vec{E}_{III} = \frac{\sigma}{\epsilon_0} \hat{n}$$

$$(4) \vec{E}_I = 0, \vec{E}_{II} = \frac{\sigma}{\epsilon_0} \hat{n}, \vec{E}_{III} = 0$$

**Sol. (3)**



$$\therefore E_I = -\frac{\sigma}{E_0} \hat{n}$$

$$\therefore E_{II} = 0$$

$$\therefore E_{III} = -\frac{\sigma}{E_0} \hat{n}$$

**3.** A mercury drop of radius  $10^{-3}$  m is broken into 125 equal size droplets.

Surface tension of mercury is  $0.45 \text{ Nm}^{-1}$ . The gain in surface energy is:

- (1)  $28 \times 10^{-5} \text{ J}$       (2)  $17.5 \times 10^{-5} \text{ J}$       (3)  $5 \times 10^{-5} \text{ J}$       (4)  $2.26 \times 10^{-5} \text{ J}$

**Sol. (4)**

$$[\text{Volume of bigger drop}] = [\text{volume of smaller drop}] \times 125$$

$$\frac{4}{3} \pi R^3 = 125 \times \frac{4}{3} \pi r^3$$

$$R^3 = 125r^3$$

$$\therefore R = 5 \times r$$

$$\Rightarrow \text{Gain in surface energy} = T \Delta A$$

$$= 0.45 \times [A_2 - A_1]$$

$$= 0.45 \times [125 \times 4\pi r^2 - 4\pi R^2]$$

$$= 0.45 \times \left[ 125 \times 4\pi \left( \frac{R}{5} \right)^2 - 4\pi R^2 \right]$$

$$= 0.45 \times [20\pi R^2 - 4\pi R^2]$$

$$= 0.45 \times 16\pi R^2$$

$$= 0.45 \times 16 \times 3.14 \times (10^{-3})^2$$

$$= 2.26 \times 10^{-5} \text{ J}$$

**4.** If earth has a mass nine times and radius twice to that of a planet P. Then  $\frac{v_e}{3} \sqrt{x} \text{ ms}^{-1}$  will be the minimum velocity required by a rocket to pull out of gravitational force of P, where  $v_e$  is escape velocity on earth. The value of  $x$  is

- (1) 1      (2) 3      (3) 18      (4) 2

**Sol. (4)**

$$M_E = 9M_p$$

$$R_E = 2R_p$$

$$\begin{aligned} V_c^1 &= \sqrt{\frac{2GM_p}{R_p}} = \sqrt{\frac{2G \frac{M_E}{9}}{\frac{R_E}{2}}} \\ &= \sqrt{\frac{2GM_E}{R_E}} \times \sqrt{\frac{2}{9}} \\ \boxed{V_c^1 &= \frac{V_e}{3} \sqrt{2}} \end{aligned}$$

**5.** A sample of gas at temperature  $T$  is adiabatically expanded to double its volume. The work done by the gas in the process is  $\left( \text{given, } \gamma = \frac{3}{2} \right)$  :

$$(1) W = \frac{T}{R} [\sqrt{2} - 2] \quad (2) W = RT[2 - \sqrt{2}] \quad (3) W = TR[\sqrt{2} - 2] \quad (4) W = \frac{R}{T} [2 - \sqrt{2}]$$

**Sol. (2)**

Work done in the process is given by

$$W = \frac{R}{\gamma - 1} (T_1 - T_2)$$

For adiabatic process:

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T V_1^{\frac{3}{2}-1} = T_2 (2V)^{\frac{3}{2}-1}$$

$$T V^{\frac{1}{2}} = T_2 (2V)^{\frac{1}{2}}$$

$$T^2 V = T_2^2 \times 2V$$

$$\therefore T_2 = \frac{T}{\sqrt{2}}$$

$$\therefore W = \frac{R}{\gamma - 1} \times \left( T - \frac{T}{\sqrt{2}} \right)$$

$$= 2RT \left[ 1 - \frac{1}{\sqrt{2}} \right]$$

$$= RT \left[ 2 - \frac{2}{\sqrt{2}} \right]$$

$$= RT[2 - \sqrt{2}]$$

$$\boxed{W = RT[2 - \sqrt{2}]}$$

6.  $\left(P + \frac{a}{V^2}\right)(V - b) = RT$  represents the equation of state of some gases. Where  $P$  is the pressure,  $V$  is the volume,  $T$  is the temperature and  $a, b, R$  are the constants. The physical quantity, which has dimensional formula as that of  $\frac{b^2}{a}$ , will be:
- (1) Compressibility (2) Energy density  
(3) Modulus of rigidity (4) Bulk modulus

**Sol. (1)**

$$[b] = [L^3]$$

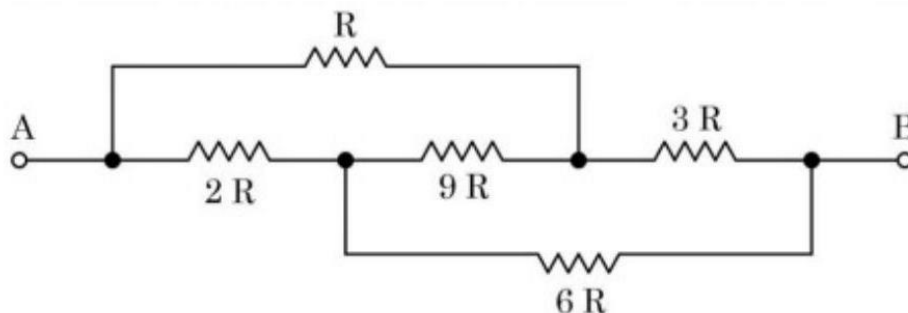
$$[a] = [PV^2]$$

$$= [ML^{-1}T^{-2}][L^6]$$

$$= [ML^5T^{-2}]$$

$$\frac{[b^2]}{[a]} = \frac{[L^6]}{[ML^5T^{-2}]} = [M^{-1}L^1T^2]$$

7. The equivalent resistance between  $A$  and  $B$  of the network shown in figure:



(1)  $\frac{8}{3}R$

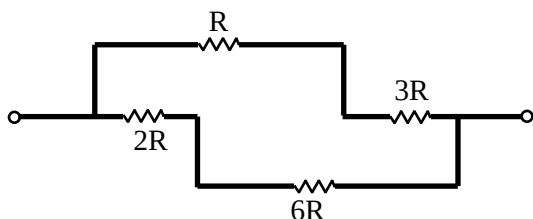
(2)  $21R$

(3)  $14R$

(4)  $11\frac{2}{3}R$

**Sol. (1)**

$\therefore$  The given network is wheat-stone network



$$\therefore R_{eq} = \frac{4R \times 8R}{4R + 8R}$$

$$= \frac{4R \times 8R}{12R}$$

$$R_{eq} = \frac{8}{3}R$$

8. Match List I with List II:

| List I                           | List II                       |
|----------------------------------|-------------------------------|
| A. AC generator                  | I. Presence of both L and C   |
| B. Transformer                   | II. Electromagnetic Induction |
| C. Resonance phenomenon to occur | III. Quality factor           |
| D. Sharpness of resonance        | IV. Mutual Induction          |

Choose the correct answer from the options given below:

(1) A-IV, B-III, C-I, D-II

(2) A-IV, B-II, C-I, D-III

(3) A-II, B-IV, C-I, D-III

(4) A-II, B-I, C-III, D-IV

**Sol. (3)**

(A) A.C. generator  $\rightarrow$  II. Electro-magnetic induction

(B) transformer  $\rightarrow$  IV Mutual induction

(C) Resonance phenomenon to occur  $\rightarrow$  (I) presence of both L and C

(D) Sharpness of resonance  $\rightarrow$  (III) Quality factor

9. An object moves with speed  $v_1, v_2$  and  $v_3$  along a line segment AB, BC and CD respectively as shown in figure. Where  $AB = BC$  and  $AD = 3AB$ , then average speed of the object will be:



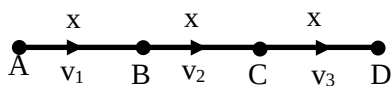
(1)  $\frac{(v_1 + v_2 + v_3)}{3v_1v_2v_3}$

(2)  $\frac{(v_1 + v_2 + v_3)}{3}$

(3)  $\frac{3v_1v_2v_3}{(v_1v_2 + v_2v_3 + v_3v_1)}$

(4)  $\frac{v_1v_2v_3}{3(v_1v_2 + v_2v_3 + v_3v_1)}$

**Sol. (3)**



$$\langle v \rangle = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}}$$

$$= \frac{3}{\left[ \frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} \right]} = \frac{3}{\left[ \frac{v_2v_3 + v_1v_3 + v_1v_2}{v_1v_2v_3} \right]}$$

$$\langle v \rangle = \frac{3v_1v_2v_3}{[v_3v_2 + v_1v_2 + v_1v_3]}$$

- 10.** 'n' polarizing sheets are arranged such that each makes an angle  $45^\circ$  with the preceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be  $I/64$ . The value of n will be:

(1) 4 (2) 3 (3) 5 (4) 6

**Sol. (D)**

According to Malus law:

$$I = \frac{I_0}{2} \left[ \cos^2 45^\circ \times \cos^2 45^\circ \times \cos^2 45^\circ \times \dots (n-1) \text{ times} \right]$$

$$\frac{I_0}{64} = \frac{I_0}{2} \times \left( \frac{1}{2} \right)^{n-1}$$

$$\frac{1}{32} = \frac{1}{2^{n-1}} \Rightarrow \frac{1}{(2)^5} = \frac{1}{2^{n-1}}$$

$$\therefore n - 1 = 5$$

$$\therefore n = 6$$

- 11.** Match List I with List II:

| List I         | List II                                                        |
|----------------|----------------------------------------------------------------|
| A. Microwaves  | I. Radio active decay of the nucleus                           |
| B. Gamma rays  | II. Rapid acceleration and deceleration of electron in aerials |
| C. Radio waves | III. Inner shell electrons                                     |
| D. X-rays      | IV. Klystron valve                                             |

Choose the correct answer from the options given below:

- (1) A-I, B-III, C-IV, D-II (2) A-IV, B-I, C-II, D-III  
(3) A-IV, B-III, C-II, D-I (4) A-I, B-II, C-III, D-IV

**Sol. (B)**

- (A) Micro-wave (IV) Klystron valve  
(B) Gamma rays (I) Radio-active decay of nucleus  
(C) Radio-waves (II) Rapid acceleration and deceleration of electrons in aerials  
(D) X-rays (III) Inner shell electron

- 12.** A proton moving with one tenth of velocity of light has a certain de Broglie wavelength of  $\lambda$ . An alpha particle having certain kinetic energy has the same de-Broglie wavelength  $\lambda$ . The ratio of kinetic energy of proton and that of alpha particle is:

(1) 2: 1 (2) 1: 2 (3) 1: 4 (4) 4: 1

**Sol. (C)**

The wavelength of matter is given by

$$\lambda = \frac{h}{p}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{2k_\alpha m_\alpha}}{\sqrt{2k_p m_p}} = 1$$

$$\therefore \frac{k_\alpha}{k_p} \times \frac{m_\alpha}{m_p} = 1 \Rightarrow \frac{k_\alpha}{k_p} = \frac{m_p}{m_\alpha}$$

$$\frac{k_\alpha}{k_p} = \frac{1}{4}$$

- 13.** A block of mass 5 kg is placed at rest on a table of rough surface. Now, if a force of 30 N is applied in the direction parallel to surface of the table, the block slides through a distance of 50 m in an interval of time 10 s. Coefficient of kinetic friction is (given,  $g = 10 \text{ ms}^{-2}$ ):

- (1) 0.60                      (2) 0.25                      (3) 0.75                      (4) 0.50

**Sol. (D)**

$$S = ut + \frac{1}{2}at^2$$

$$50 = 0 \times t + \frac{1}{2} \times a \times (10)^2$$

$$50 = \frac{1}{2} \times a \times 100$$

$$a = \frac{100}{100} \Rightarrow \boxed{a = 1 \text{ m/s}^2}$$

$$\sum F_x = ma_x$$

$$30 - \mu mg = ma$$

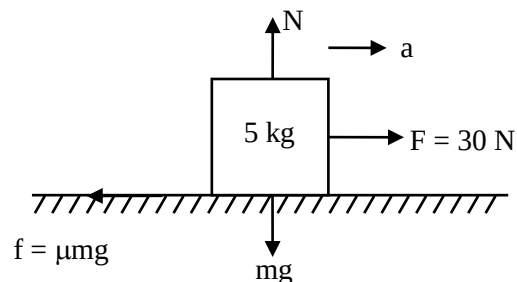
$$30 - \mu \times 50 = 5$$

$$50\mu = 25$$

$$\mu = \frac{25}{50}$$

$$= \frac{1}{2}$$

$$\Rightarrow \boxed{\mu = 0.5}$$



- 14.** Given below are two statements:

**Statement I:** Acceleration due to gravity is different at different places on the surface of earth.

**Statement II:** Acceleration due to gravity increases as we go down below the earth's surface.

In the light of the above statements, choose the correct answer from the options given below

- (1) Statement I is false but Statement II is true  
 (2) Statement I is true but Statement II is false  
 (3) Both Statement I and Statement II are false  
 (4) Both Statement I and Statement II are true

**Sol. (B)**

Statement (I) is true But  
Statement (II) is false

**15.** Which of the following frequencies does not belong to FM broadcast.

- (1) 64MHz (2) 89MHz (3) 99MHz (4) 106MHz

**Sol. (A)**

The Frequencies for FM Broadcast is between 87.5 MHz to 108 MHz.

**16.** The mass of proton, neutron and helium nucleus are respectively  $1.0073u$ ,  $1.0087u$  and  $4.0015u$ . The binding energy of helium nucleus is:

- (1) 28.4MeV (2) 56.8MeV (3) 14.2MeV (4) 7.1MeV

**Sol. (A)**

$$2P + 2n = {}^4_2\text{He} + E$$

$$\begin{aligned}\therefore \text{B.E} &= [2 \times (1.0073 + 1.0087) - 4.0015] \times 931 \\ &= 0.0305 \times 931 \\ &= 28.3955 \text{ MeV}\end{aligned}$$

**17.** A steel wire with mass per unit length  $7.0 \times 10^{-3} \text{ kg m}^{-1}$  is under tension of 70 N. The speed of transverse waves in the wire will be:

- (1) 100 m/s (2) 10 m/s (3) 50 m/s (4)  $200\pi \text{ m/s}$

**Sol. (A)**

The velocity of Transverse wave on string is given by

$$\begin{aligned}V &= \sqrt{\frac{T}{\mu}} \\ &= \sqrt{\frac{70}{7 \times 10^{-3}}} = \sqrt{\frac{70 \times 10^3}{7}} \\ &= \sqrt{10^4} = 100 \text{ m/s}\end{aligned}$$

**18.** Match List I with List II:

| List I                     | List II                                                        |
|----------------------------|----------------------------------------------------------------|
| A. Intrinsic semiconductor | I. Fermi-level near the valence band                           |
| B. n-type semiconductor    | II. . Fermi-level in the middle of valence and conduction band |
| C. p-type semiconductor    | III. Fermi-level near the conduction band                      |
| D. Metals                  | IV. Fermi-level inside the conduction band                     |

Choose the correct answer from the options given below:

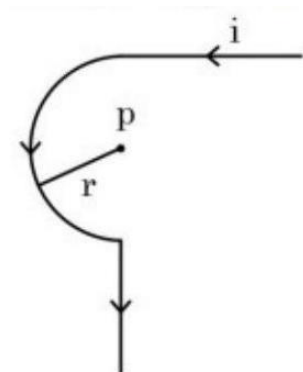
- (1) A-II, B-III, C-I, D-IV (2) A-I, B-II, C-III, D-IV  
(3) A-II, B-I, C-III, D-IV (4) A-III, B-I, C-II, D-IV



**Sol. (A)**

- |                          |                                                               |
|--------------------------|---------------------------------------------------------------|
| (A) Intrinsic            | (II) Fermi-level in the middle of valence and conduction band |
| (B) n-type semiconductor | (III) Fermi-level near conduction band                        |
| (C) p-type semiconductor | (I) Fermi-level near valence band                             |
| (D) Metals               | (IV) Fermi-level inside the conduction band                   |

- 19.** Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires.



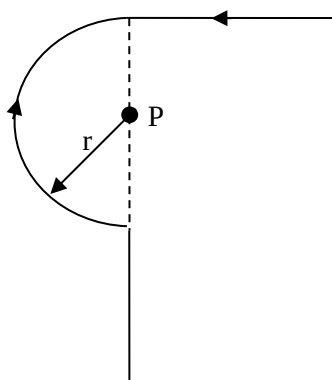
(1)  $\frac{\mu_0 i}{2r} \left(1 + \frac{2}{\pi}\right)$

(2)  $\frac{\mu_0 i}{2r} \left(\frac{1}{2} + \frac{1}{2\pi}\right)$

(3)  $\frac{\mu_0 i}{2r} \left(1 + \frac{1}{\pi}\right)$

(4)  $\frac{\mu_0 i}{2r} \left(\frac{1}{2} + \frac{1}{\pi}\right)$

**Sol. (B)**



$$B_P = B_1 + B_2$$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r}$$

$$= \frac{\mu_0 i}{4r} \left[ \frac{1}{\pi} + 1 \right]$$

$$\boxed{B_P = \frac{\mu_0 i}{2r} \left[ \frac{1}{2\pi} + \frac{1}{2} \right]}$$

- 20.** The average kinetic energy of a molecule of the gas is

- |                                          |                                        |
|------------------------------------------|----------------------------------------|
| (1) proportional to absolute temperature | (2) proportional to pressure           |
| (3) proportional to volume               | (4) dependent on the nature of the gas |

**Sol. (A)**

The average kinetic energy of gas molecule is given by,

$$K.E_{avg} = \frac{3}{2} KT$$

$$\boxed{\therefore K.E_{avg} \propto T}$$

## SECTION - B

- 21.** A small particle moves to position  $5\hat{i} - 2\hat{j} + \hat{k}$  from its initial position  $2\hat{i} + 3\hat{j} - 4\hat{k}$  under the action of force  $5\hat{i} + 2\hat{j} + 7\hat{k}$  N. The value of work done will be \_\_\_\_\_ J.

**Sol.** 40

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$= (5\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k})$$

$$\Delta \vec{r} = 3\hat{i} - 5\hat{j} + 5\hat{k}$$

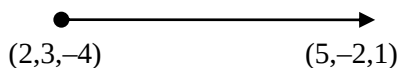
$$\therefore W = \vec{F} \cdot \Delta \vec{r}$$

$$= (5\hat{i} + 2\hat{j} + 7\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 5\hat{k})$$

$$= 15 - 10 + 35$$

$$= 5 + 35$$

$$\boxed{W = 40J}$$



- 22.** A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. The ratio of Bulk modulus of water to that of the liquid is  $\frac{3}{x}$ .

The value of  $x$  is \_\_\_\_\_.

**Sol.** 1

$$\text{Bulk Modulus} = V \frac{dP}{dV}$$

$$\frac{(B)_{\text{water}}}{(B)_{\text{liquid}}} = \frac{V dP / dV}{V dP / dV} = \frac{dP / 0.01}{dP / 0.03}$$

$$\therefore \frac{(B)_{\text{water}}}{(B)_{\text{liquid}}} = \frac{0.03}{0.01} = \frac{3}{1}$$

$$\boxed{\frac{(B)_{\text{water}}}{(B)_{\text{liquid}}} = \frac{3}{1}}$$

$\therefore$  On comparing with  $\frac{3}{x}$ , The value of "x" will be "1".

- 23.** A light of energy 12.75 eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The angular momentum of the atom in the excited state is  $\frac{x}{\pi} \times 10^{-17}$  eVs. The value of  $x$  is \_\_\_\_\_ (use  $h = 4.14 \times 10^{-15}$  eVs,  $c = 3 \times 10^8$  ms<sup>-1</sup>).

**Sol.**  $x = 828$

The energy of electron in ground state = -13.6 eV

$$E_n - E_1 = 12.75$$

$$\therefore E_n = 12.75 - 13.6$$

$$E_n = -0.85$$

So "n" is given by

$$E_n = -\frac{13.6}{n^2}$$

$$n^2 = \frac{-13.6}{-0.85}$$

$$n^2 = 16 \Rightarrow \boxed{n = 4}$$

$$\Rightarrow L = \frac{nh}{2\pi} = \frac{x}{\pi} \times 10^{-17}$$

$$\Rightarrow 4 \times \frac{h}{2\pi} = \frac{x}{\pi} \times 10^{-17}$$

$$4 \times \frac{4.14 \times 10^{-15}}{2\pi} = \frac{x}{\pi} \times 10^{-17} \Rightarrow \frac{2 \times 4.14 \times 10^{-15}}{10^{-17}} = x$$

$$x = 8.28 \times 10^2 \Rightarrow \boxed{x = 828}$$

- 24.** A charge particle of  $2\mu\text{C}$  accelerated by a potential difference of 100 V enters a region of uniform magnetic field of magnitude 4mT at right angle to the direction of field. The charge particle completes semicircle of radius 3 cm inside magnetic field. The mass of the charge particle is \_\_\_\_\_  $\times 10^{-18}$  kg.

**Sol.** 144

$$R = \frac{mv}{qB} = \frac{p}{qB}$$

$$R = \frac{\sqrt{2mq\Delta V}}{qB}$$

$$3 \times 10^{-2} = \frac{\sqrt{2m \times 2 \times 10^{-6} \times 10^2}}{2 \times 10^{-6} \times 4 \times 10^{-3}}$$

$$3 \times 10^{-2} \times 2 \times 10^{-6} \times 4 \times 10^{-3} = \sqrt{4m \times 10^{-4}}$$

$$24 \times 10^{-11} = \sqrt{4m \times 10^{-4}}$$

$$m = \frac{24 \times 24 \times 10^{-22}}{4 \times 10^{-4}}$$

$$\boxed{m = 144 \times 10^{-18} \text{ Kg}}$$

- 25.** The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is: \_\_\_\_\_ cm.

**Sol.** 2

$$\text{K.E} = \text{P.E} + \frac{25}{100} \times \text{P.E.}$$

$$\text{K.E} = \text{P.E} + \frac{1}{4} \text{P.E}$$

$$\text{K.E} = \frac{5}{4} \text{P.E}$$

$$\frac{1}{2} K (A^2 - x^2) = \frac{5}{4} \times \frac{1}{2} K x^2$$

$$4(A^2 - x^2) = 5x^2$$

$$4A^2 - 4x^2 = 5x^2$$

$$9x^2 = 4A^2$$

$$x^2 = \frac{4}{9} \times (3)^2$$

$$\boxed{\therefore x = \pm 2}$$

- 26.** In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf  $E$ , the length-of null point increases by 40 cm. The value of  $E$  is  $\frac{x}{10}$  V. The value of  $x$  is \_\_\_\_\_.

**Sol.** 25

$$E_1 = K\ell_1 \quad \dots(i)$$

$$E_2 = K\ell_2 \quad \dots(ii)$$

$$\therefore \frac{E_2}{E_1} = \frac{\ell_2}{\ell_1}$$

$$\frac{E}{1.5} = \frac{100}{60}$$

$$\therefore E = 1.5 \times \frac{10}{6}$$

$$= \frac{3}{2} \times \frac{10}{6}$$

$$= \frac{5}{2}$$

$$= 2.5$$

$$= \frac{25}{10}$$

$$\boxed{\therefore x = 25}$$

- 27.** A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be  $\frac{x}{3}$  cm. The value of  $x$  is \_\_\_\_\_.

**Sol.** 32

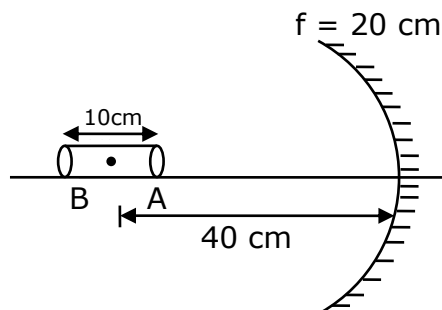


Image of end A:

$$u = -35 \text{ cm}$$

$$f = -20 \text{ cm}$$

$$v = ?$$

$$v = \frac{uf}{u - f}$$

$$= \frac{-35 \times -20}{-35 + 20}$$

$$= \frac{-35 \times -20}{-15}$$

$$v = -\frac{140}{3}$$

Image of end B:

$$u = -45 \text{ cm}$$

$$v = ?$$

$$f = -20 \text{ cm}$$

$$v = \frac{uf}{u - f}$$

$$= \frac{-45 \times -20}{-45 + 20}$$

$$= \frac{-45 \times -20}{-25}$$

$$v = -36$$

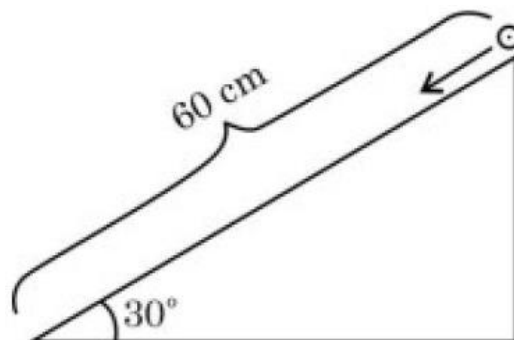
$$\therefore \text{length of image} = \left| -36 + \frac{140}{3} \right|$$

$$= \left| -\frac{108 + 140}{3} \right|$$

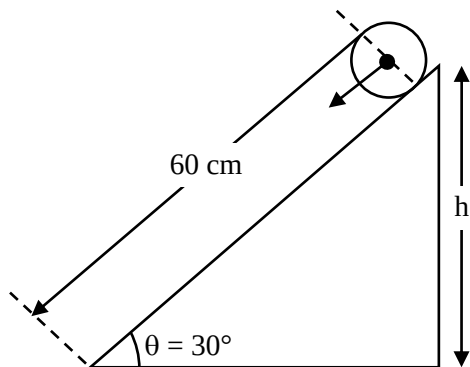
$$= \frac{32}{3}$$

$\therefore$  The value of  $x = 32$

- 28.** A solid cylinder is released from rest from the top of an inclined plane of inclination  $30^\circ$  and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is \_\_\_\_\_  $\text{ms}^{-1}$ .  
(Given  $g = 10 \text{ ms}^{-2}$ )



**Sol.**



$$h = 60 \sin 30$$

$$\therefore h = 30 \text{ cm}$$

The velocity of by linder upon reaching the ground is given by

$$V = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}}$$

$$\therefore V = \sqrt{\frac{2 \times 10 \times 30 \times 10^{-2}}{1 + \frac{1}{2}}}$$

$$= \sqrt{\frac{6 \times 2}{3}}$$

$$V = 2 \text{ m/s}$$

- 29.** A series LCR circuit is connected to an ac source of 220 V, 50 Hz. The circuit contain a resistance  $R = 100\Omega$  and an inductor of inductive reactance  $X_L = 79.6\Omega$ . The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be \_\_\_\_\_  $\mu\text{F}$ .

**Sol. 40**

For maximum power, the LCR must be in resonance.

$$\therefore X_L = X_C$$

$$79.6 = \frac{1}{\omega C}$$

$$C = \frac{1}{\omega \times 79.6}$$

$$= \frac{1}{2\pi \times 50 \times 79.6}$$

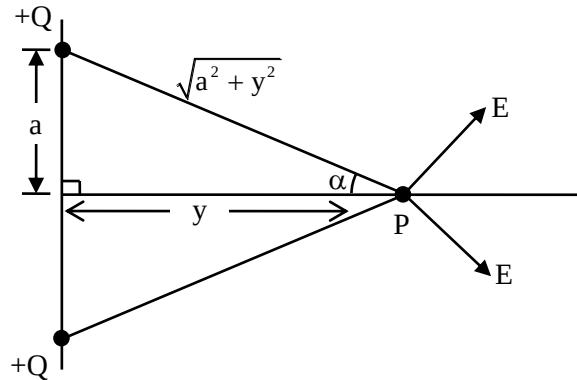
$$= \frac{1}{100\pi \times 79.6}$$

$$= 40 \times 10^{-6}$$

$$C = 40\mu\text{F}$$

- 30.** Two equal positive point charges are separated by a distance  $2a$ . The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge  $q_0$  becomes maximum is  $\frac{a}{\sqrt{x}}$ . The value of  $x$  is \_\_\_\_\_.

**Sol.** 2



Electric field at point "P" due to any one charge =  $\frac{KQ}{a^2 + y^2}$

$\therefore$  Net electric field at point "P" will be

$$E_{\text{net}} = 2E \cos \alpha$$

$$= \frac{2KQ}{a^2 + y^2} \times \frac{y}{\sqrt{a^2 + y^2}}$$

$$E_{\text{net}} = \frac{2KQy}{(a^2 + y^2)^{3/2}}$$

$\Rightarrow$  Electric force (F) =  $E_{\text{net}} q_0$

$$= \frac{2K Q q_0 y}{(a^2 + y^2)^{3/2}}$$

For  $F = \text{max} \Rightarrow \frac{dF}{dy} = 0$

By solving, we get  $y = \frac{a}{\sqrt{2}}$

$\therefore$  the value of  $x = 2$

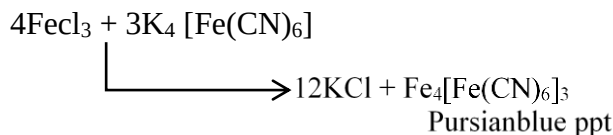
# Chemistry

## SECTION - A

31. A solution of  $\text{FeCl}_3$  when treated with  $\text{K}_4[\text{Fe}(\text{CN})_6]$  gives a prussian blue precipitate due to the formation of

(1)  $\text{K}[\text{Fe}_2(\text{CN})_6]$  (2)  $\text{Fe}_4[\text{Fe}(\text{CN})_6]_3$  (3)  $\text{Fe}[\text{Fe}(\text{CN})_6]$  (4)  $\text{Fe}_3[\text{Fe}(\text{CN})_6]_2$

Sol. 2



32. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R  
**Assertion A:** Hydrogen is an environment friendly fuel.

**Reason R:** Atomic number of hydrogen is 1 and it is a very light element.

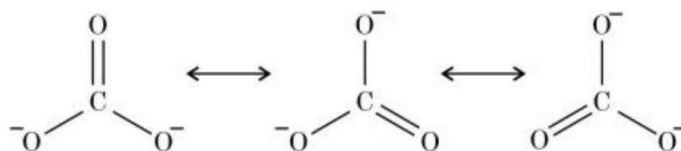
In the light of the above statements, choose the correct answer from the options given below

- (1) A is true but R is false  
 (2) A is false but R is true  
 (3) Both A and R are true and R is the correct explanation of A  
 (4) Both A and R are true but R is NOT the correct explanation of A

Sol. 4

No pollution occurs by combustion of hydrogen and very low density of hydrogen.

33. Resonance in carbonate ion ( $\text{CO}_3^{2-}$ ) is



Which of the following is true?

- (1) All these structures are in dynamic equilibrium with each other.  
 (2) It is possible to identify each structure individually by some physical or chemical method.  
 (3) Each structure exists for equal amount of time.  
 (4)  $\text{CO}_3^{2-}$  has a single structure i.e., resonance hybrid of the above three structures.

Sol. 4

Resonating structure are hypothtical and resonance hybrid is a real structure which is weighted average of all the resonating struture.

34. Match List I with List II

| List I            | List II                    |
|-------------------|----------------------------|
| (A) Tranquilizers | (I) Anti blood clotting    |
| (B) Aspirin       | (II) Salvarsan             |
| (C) Antibiotic    | (III) antidepressant drugs |
| (D) Antiseptic    | (IV) soframicine           |

Choose the correct answer from the options given below:

- (1) (A) – IV, (B) – II, (C) – I, (D) – III (2) (A) – II, (B) – I, (C) – III, (D) – IV  
 (3) (A) - III, (B) - I, (C) - II, (D) – IV (4) (A) - II, (B) – IV, (C) – I, (D) – III



**Sol. 3**

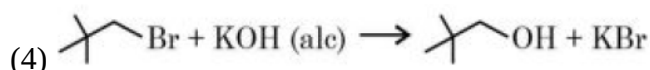
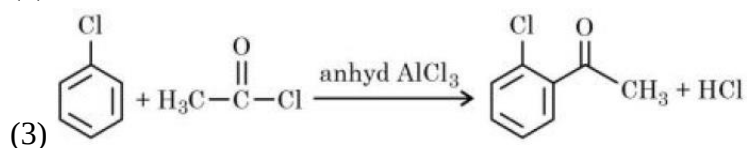
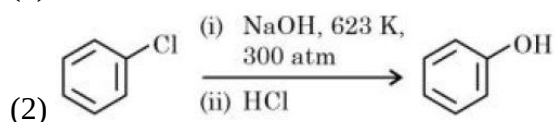
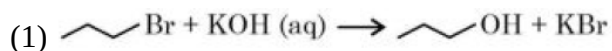
A → (iii)

B → (i)

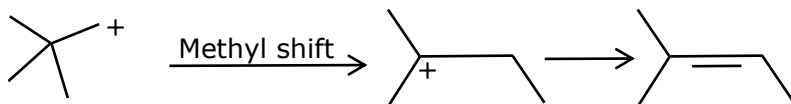
C → (ii)

D → (iv)

**35.** Identify the incorrect option from the following:

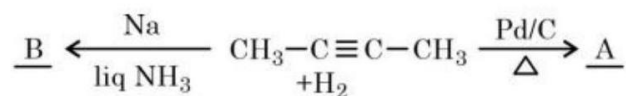


**Sol. 4**



In question given option reaction is incorrect so right answer is (4)

**36.** But-2-yne is reacted separately with one mole of Hydrogen as shown below:



A. A is more soluble than B.

B. The boiling point & melting point of A are higher and lower than B respectively.

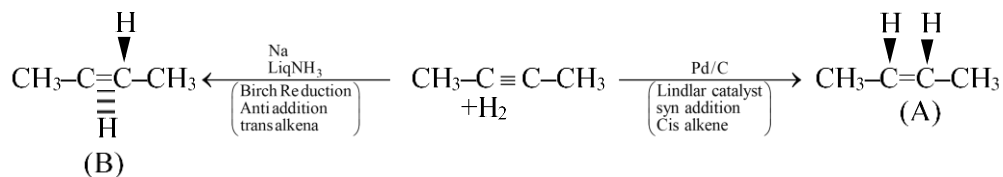
C. A is more polar than B because dipole moment of A is zero.

D. Br<sub>2</sub> adds easily to B than A.

Identify the incorrect statements from the options given below:

(1) B, C & D only    (2) A and B only    (3) A, C & D only    (4) B and C only

**Sol. 2**



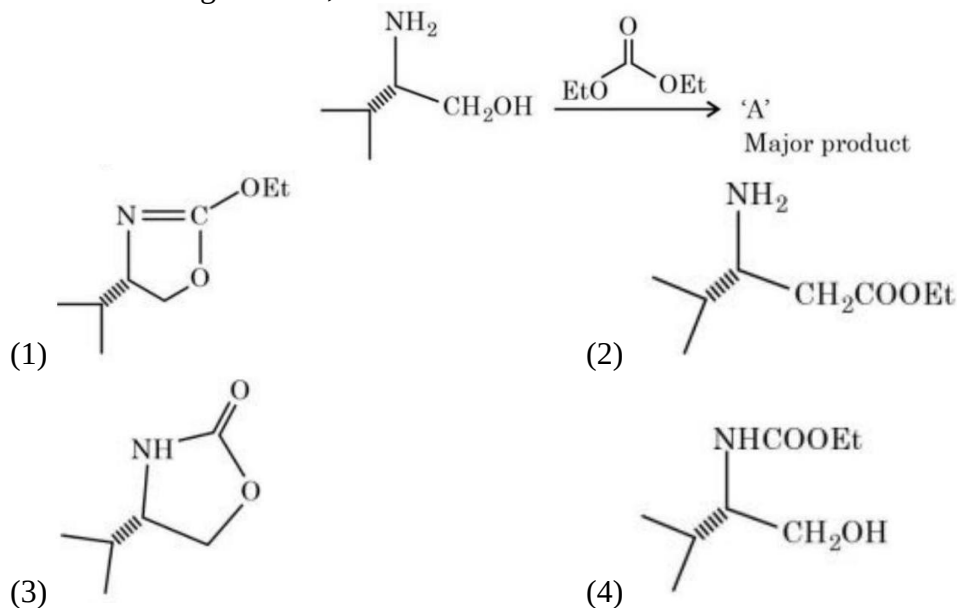
A) Cis has dipole moment, more soluble than trans (B)

B) B.P.(cis > trans), M.P. (trans > cis)

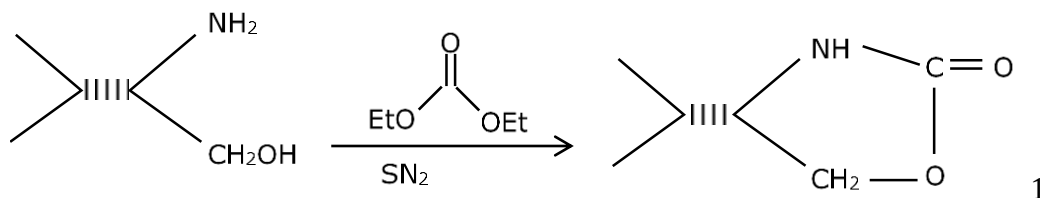
C) Dipole moment (A > B) but  $\mu_A \neq 0$

D) Br<sub>2</sub> add easily to A not B

37. In the following reaction, 'A' is



Sol.



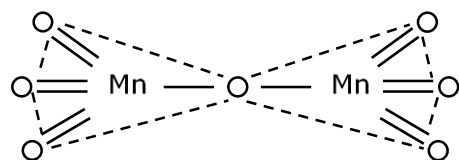
38. Highest oxidation state of Mn is exhibited in Mn<sub>2</sub>O<sub>7</sub>. The correct statements about Mn<sub>2</sub>O<sub>7</sub> are

- (A) Mn is tetrahedrally surrounded by oxygen atoms.  
 (B) Mn is octahedrally surrounded by oxygen atoms.  
 (C) Contains Mn-O-Mn bridge.  
 (D) Contains Mn-Mn bond.

Choose the correct answer from the options given below:

- (1) A and C only      (2) A and D only      (3) B and C only      (4) B and D only

Sol. 1 (A & C)



39. Match List I with List II

|     | List I             | List II                                                    |
|-----|--------------------|------------------------------------------------------------|
| (A) | Slaked lime        | (I) NaOH                                                   |
| (B) | Dead burnt plaster | (II) Ca(OH) <sub>2</sub>                                   |
| (C) | Caustic soda       | (III) Na <sub>2</sub> CO <sub>3</sub> · 10H <sub>2</sub> O |
| (D) | Washing soda       | (IV) CaSO <sub>4</sub>                                     |

Choose the correct answer from the options given below:

- (1) (A) - III, (B) - IV, (C) - II, (D) - I      (2) (A) - III, (B) - II, (C) - IV, (D) - I  
 (3) (A) - I, (B) - IV, (C) - II, (D) - III      (4) (A) - II, (B) - IV, (C) - I, (D) - III

**Sol. 4**

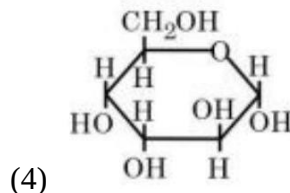
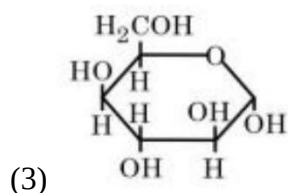
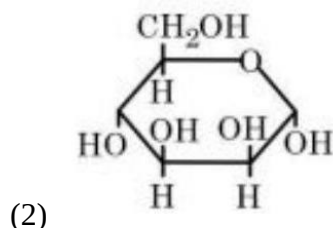
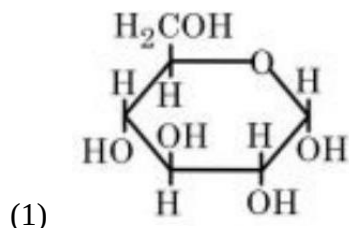
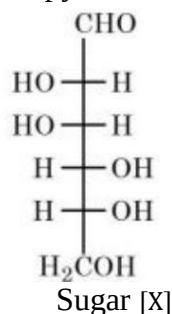
Slaked Lime  $\rightarrow \text{Ca(OH)}_2$

Dead burnt plaster  $\rightarrow \text{CaSO}_4$

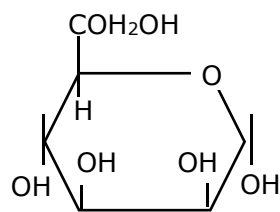
Caustic Soda  $\rightarrow \text{NaOH}$

Washing Soda  $\rightarrow \text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$

**40.** The correct representation in six membered pyranose form for the following sugar [X] is



**Sol. 2**



Haworth structure of mannose

**41.** Which of the following complex will show largest splitting of d-orbitals ?

- (1)  $[\text{FeF}_6]^{3-}$       (2)  $[\text{Fe}(\text{C}_2\text{O}_4)_3]^{3-}$       (3)  $[\text{Fe}(\text{CN})_6]^{3-}$       (4)  $[\text{Fe}(\text{NH}_3)_6]^{3+}$

**Sol. 3**

(M) Strong field ligands will split 'd' orbital largely.

$\text{CN}^-$  is SF.L      Where as  $\text{F}^-$ ,  $\text{C}_2\text{O}_4^{2-}$  &  $\text{NH}_3$

Are comparatively weak field ligand as common to  $\text{CN}^-$

**42.** Which of the following are the example of double salt?

(A)  $\text{FeSO}_4 \cdot (\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$

(B)  $\text{CuSO}_4 \cdot 4\text{NH}_3 \cdot \text{H}_2\text{O}$

(C)  $\text{K}_2\text{SO}_4 \cdot \text{Al}_2(\text{SO}_4)_3 \cdot 24\text{H}_2\text{O}$

(D)  $\text{Fe}(\text{CN})_2 \cdot 4\text{KCN}$

Choose the correct answer

(1) B and D only

(2) A and C only

(3) A and B only

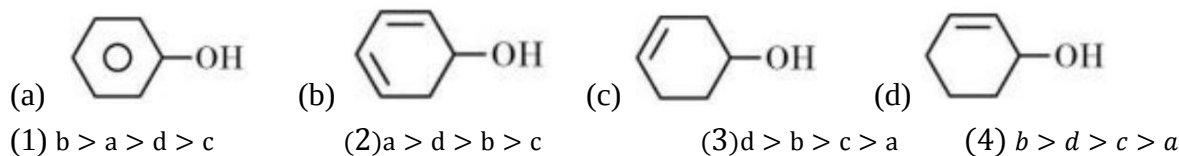
(4) A, B and D only

**Sol. 1**

Double salt contains two or more types of salts.

$\text{CuSO}_4 \cdot 4\text{NH}_3 \cdot \text{H}_2\text{O}$  and  $\text{Fe}(\text{CN})_2 \cdot 4\text{KCN}$  are complex compounds.

**43.** Decreasing order of dehydration of the following alcohols is



**Sol. 4**

Ease of hydration  $\propto$  stability of carbocation

$b > d > c > a$

**44.** Given below are two statements:

**Statement I:** Chlorine can easily combine with oxygen to form oxides; and the product has a tendency to explode.

**Statement II:** Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both the Statements I and II are true  
(2) Both the Statements I and II are false  
(3) Statement I is false but Statement II is true  
(4) Statement I is true but Statement II is false

**Sol. 1**

Chlorine oxides,  $\text{Cl}_2\text{O}$ ,  $\text{ClO}_2$ ,  $\text{Cl}_2\text{O}_6$  and  $\text{Cl}_2\text{O}_7$  are highly Reactive oxidising Agents and tend to explode.

**45.** Choose the correct statement(s):

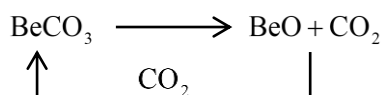
- A. Beryllium oxide is purely acidic in nature.  
B. Beryllium carbonate is kept in the atmosphere of  $\text{CO}_2$ .  
C. Beryllium sulphate is readily soluble in water.  
D. Beryllium shows anomalous behavior.

Choose the correct answer from the options given below:

- (1) B, C and D only (2) A only (3) A, B and C only (4) A and B only

**Sol. 1**

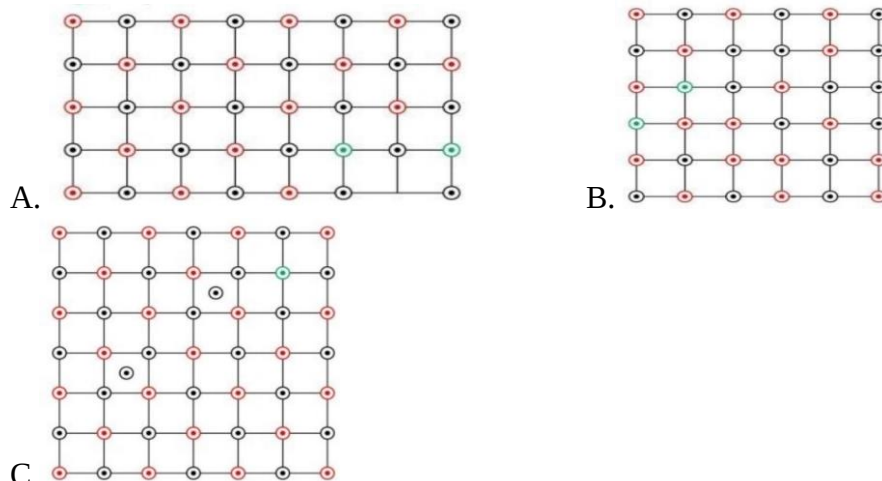
$\text{BeO}$  is Amphoteric



$\text{BeSO}_4$  is soluble in water

Due to small size Be shows anomalous behaviour.

46. Which of the following represents the lattice structure of  $A_{0.95}O$  containing  $A^{2+}$ ,  $A^{3+}$  and  $O^{2-}$  ions?  
 $\odot A^{2+}$   $\odot A^{3+}$   $\odot O^{2-}$



- (1) A only                      (2) B and C only                      (3) A and B only                      (4) B only

**Sol. 1**

Some vacancy generated by this type defect.

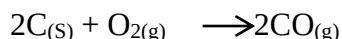
47. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R  
**Assertion A:** In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.

**Reason R:** CO tends to get decomposed at higher temperature.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both A and R are correct but R is NOT the correct explanation of A  
 (2) Both A and R are correct and R is the correct explanation of A  
 (3) A is correct but R is not correct  
 (4) A is not correct but R is correct

**Sol. 3**



$$\Delta S^\circ \text{ is the, } \Delta G^\circ = \Delta H^\circ - T \Delta S$$

Thus slope is Negative.

As temperature Increase  $\Delta C$  becomes more Negative thus it has loner tendency to get decomposed.

48. Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R  
**Assertion A:** Amongst He, Ne, Ar and Kr; 1 g of activated charcoal adsorbs more of Kr.

**Reason R:** The critical volume  $V_c$  ( $\text{cm}^3 \text{mol}^{-1}$ ) and critical pressure  $P_c$  (atm) is highest for Krypton but the compressibility factor at critical point  $Z_c$  is lowest for Krypton.

In the light of the above statements, choose the correct answer from the options given below

- (1) A is true but R is false  
 (2) Both A and R are true and R is the correct explanation of A  
 (3) A is false but R is true  
 (4) Both A and R are true but R is NOT the correct explanation of A

**Sol. 1**

Assertion A correct but Reason is wrong.

49. Match List I with List II

| List I               | List II                              |
|----------------------|--------------------------------------|
| Test                 | Functional group / Class of Compound |
| (A) Molisch's Test   | (I) Peptide                          |
| (B) Biuret Test      | (II) Carbohydrate                    |
| (C) Carbylamine Test | (III) Primary amine                  |
| (D) Schiff's Test    | (IV) Aldehyde                        |

Choose the correct answer from the options given below:

(1) (A) – III, (B) – IV, (C) – I, (D) – II

(2) (A) –II, (B) – I, (C) – III, (D) - IV

(3) (A) –III, (B) – IV, (C) – II, (D) – I

(4) (A) –I, (B) – II, (C) – III, (D) – IV

**Sol. 2**

A → (II)                      C → (III)

B → (I)                        D → (IV)

50. How can photochemical smog be controlled?

(1) By using catalytic convertors in the automobiles/industry.

(2) By complete combustion of fuel.

(3) By using tall chimneys.

(4) By using catalyst.

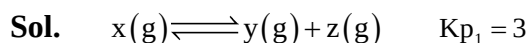
**Sol. 1**

1) By using catalytic convertors in the automobiles / industry.

51. (i)  $X(g) \rightleftharpoons Y(g) + Z(g)$   $K_{p1} = 3$

(ii)  $A(g) \rightleftharpoons 2 B(g)$   $K_{p2} = 1$

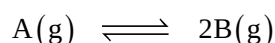
If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium  $\left(\frac{p_1}{p_2}\right)$  is equal to x : 1. The value of x is\_\_\_\_ (Nearest integer)



$$t = 0 \quad 1 \quad 0 \quad 0$$

$$t_{eq} \quad 1-x \quad x \quad x$$

$$\text{Partial Pressure} \quad \frac{(1-x)}{1+x} P_1 \quad \frac{xP_1}{1+x} \quad \frac{xP_1}{1+x}$$



$$t = 0 \quad 1 \quad 0$$

$$t_{eq} \quad 1-x \quad 2x$$

$$\text{Partial Pressure} \quad \frac{1-x}{1+x} \times P_2 \quad \frac{2x}{1+x} \times P_2$$

$$K_{p_1} = \frac{\left(\frac{xP_1}{1+x}\right)\left(\frac{xP_1}{1+x}\right)}{\left(\frac{1-x}{1+x}P_1\right)}$$

$$K_{p_2} = \frac{(2x)^2 \times P_2^2}{\left(\frac{1-x}{1+x}\right)P_2}$$

$$\frac{K_{p_1}}{K_{p_2}} = \frac{3}{1} = \frac{P_1}{4P_2}$$

$$\frac{P_1}{P_2} = \frac{12}{1}$$

52. Electrons in a cathode ray tube have been emitted with a velocity of  $1000 \text{ m s}^{-1}$ . The number of following statements which is/are true about the emitted radiation is

Given :  $h = 6 \times 10^{-34} \text{ Js}$ ,  $m_e = 9 \times 10^{-31} \text{ kg}$ .

(A) The deBroglie wavelength of the electron emitted is  $666.67 \text{ nm}$ .

(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

**Sol.** 2

$$(A) \lambda = \frac{h}{mv} = \frac{6 \times 10^{-34}}{9 \times 10^{-31} \times 1000} \\ = 666.67 \times 10^{-9} \text{ m}$$

(C) The cathode ray start from Cathode and move towards anode.

53. A and B are two substances undergoing radioactive decay in a container.

The half life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same? \_\_\_\_\_ min.

**Sol.** 15

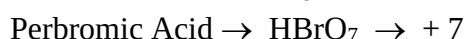
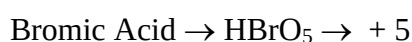
$$\text{Condition} \Rightarrow [B] = 4[A]$$

$$\text{For A} \quad A \xrightarrow[15 \text{ min}]{t_{1/2}} \frac{A}{2}$$

$$\text{For B} \quad 4A \xrightarrow[5 \text{ min}]{t_{1/2}} 2A \xrightarrow[5 \text{ min}]{t_{1/2}} A \xrightarrow[5 \text{ min}]{t_{1/2}} \frac{A}{2}$$

54. Sum of oxidation states of bromine in bromic acid and perbromic acid is

**Sol.** 12



$$\text{Sum of oxidation state} = 5 + 7 = 12$$

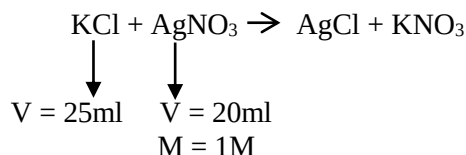
55. 25 mL of an aqueous solution of KCl was found to require 20 mL of 1M AgNO<sub>3</sub> solution when titrated using K<sub>2</sub>CrO<sub>4</sub> as an indicator. What is the depression in freezing point of KCl solutions of the given concentration? \_\_\_\_\_ (Nearest integer).

(Given: K<sub>f</sub> = 2.0 K kg mol<sup>-1</sup>)

Assume 1) 100% ionization and

2) density of the aqueous solution as 1 g mL<sup>-1</sup>

**Sol. 3**



At equivalence point,

Mmole of KCl = mmole of AgNO<sub>3</sub> = 20 mmole

Volume of solution = 25 ml

Mass of solution = 25 gm

Mass of solvent = 25 – mass of solute  
= 25 – [20 × 10<sup>-3</sup> × 74.5]  
= 23.51 gm

Molality of KCl =  $\frac{\text{mole of KCl}}{\text{mass of solvent in kg}}$

$$= \frac{20 \times 10^{-3}}{23.51 \times 10^{-3}} = 0.85$$

i of KCl = 2 (100% ionisation)

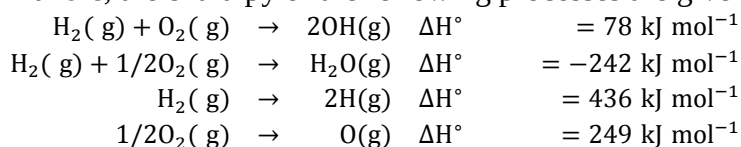
$$\Delta T_f = i \times K_f \times m$$

$$= 2 \times 2 \times 0.85$$

$$= 3.4$$

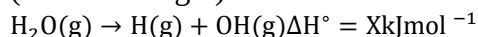
$$\approx 3$$

56. At 25°C, the enthalpy of the following processes are given:

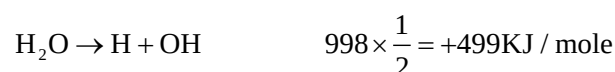
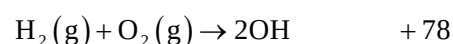
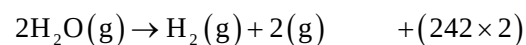


What would be the value of x for the following reaction?

(Nearest integer)



**Sol. 499**

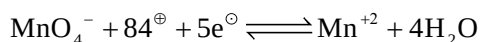




57. At what pH, given half cell  $\text{MnO}_4^- (0.1\text{M}) \mid \text{Mn}^{2+} (0.001\text{M})$  will have electrode potential of 1.282 V ? (Nearest Integer)

Given  $E_{\text{MnO}_4^-/\text{Mn}^{2+}}^0 = 1.54 \text{ V}, \frac{2.303RT}{F} = 0.059 \text{ V}$

**Sol. 3**



$$E = E^0 - \frac{0.059}{5} \log \frac{[\text{Mn}^{2+}]}{[\text{MnO}_4^-][\text{H}^+]^8}$$

$$1.282 = 1.54 - \frac{0.059}{5} \log \frac{10^{-3}}{10^{-1} \times [\text{H}^+]^8}$$

$$\frac{0.258 \times 5}{0.059} = \log \frac{10^{-2}}{[\text{H}^+]^8}$$

$$21.86 = -2 + 8\text{pH}$$

$$\text{pH} = 2.98 \approx 3$$

58. The density of 3M solution of NaCl is  $1.0 \text{ g mL}^{-1}$ . Molality of the solution is  $\times 10^{-2} \text{ m}$ . (Nearest integer).

Given: Molar mass of Na and Cl is 23 and  $35.5 \text{ g mol}^{-1}$  respectively.

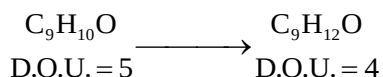
**Sol. 364**

$$m = \frac{1000 \times M}{1000d - M \times \text{M.wt}} = \frac{1000 \times 3}{1000 \times 1 - (3 \times 58.5)} = 3.64$$

$$= 364 \times 10^{-2}$$

59. Number of isomeric compounds with molecular formula  $\text{C}_9\text{H}_{10}\text{O}$  which (i) do not dissolve in NaOH (ii) do not dissolve in HCl. (iii) do not give orange precipitate with 2,4DNP (iv) on hydrogenation give identical compound with molecular formula  $\text{C}_9\text{H}_{12}\text{O}$  is

**Sol. 2**



Do not dissolve in NaOH, So no acidic group

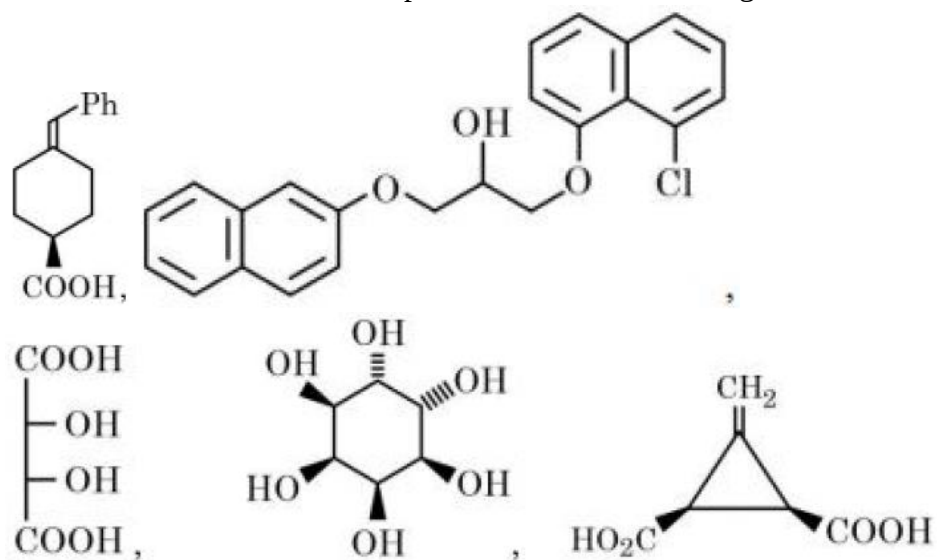
Do not dissolve in HCl, So no basic group, no alkene

Do not give orange PPT with 2, 4-DNP so no carbonyl group

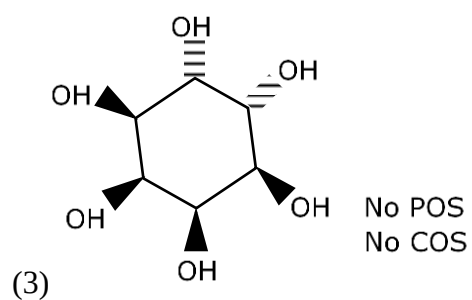
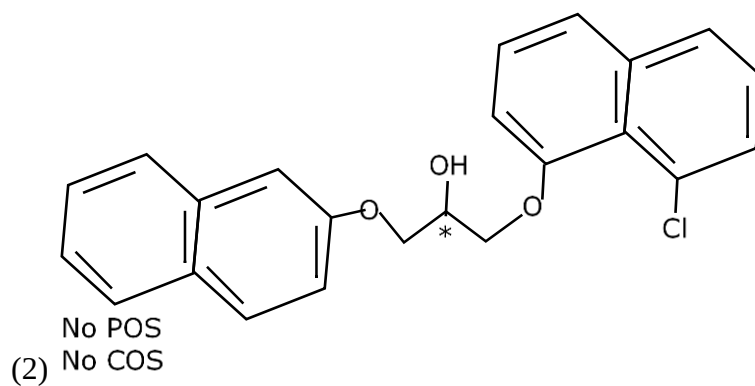
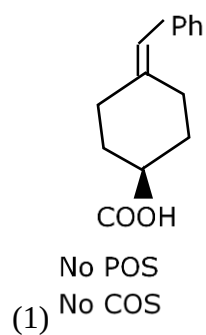
Possible compounds – cis and trans of  $\text{Ph} - \text{CH} = \text{CH} - \text{O} - \text{CH}_3$

(Also Many possible products are there)

60. The total number of chiral compound/s from the following is



Sol.



Note :- Take note from gammaxene structure

# Mathematics

## Section A

61. If  $y = y(x)$  is the solution curve of the differential equation  $\frac{dy}{dx} + y \tan x = x \sec x$ ,  $0 \leq x \leq \frac{\pi}{3}$ ,  $y(0) = 1$ , then  $y\left(\frac{\pi}{6}\right)$  is equal to

- (1)  $\frac{\pi}{12} - \frac{\sqrt{3}}{2} \log_e \left( \frac{2\sqrt{3}}{e} \right)$  (2)  $\frac{\pi}{12} - \frac{\sqrt{3}}{2} \log_e \left( \frac{2}{e\sqrt{3}} \right)$   
 (3)  $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e \left( \frac{2}{e\sqrt{3}} \right)$  (4)  $\frac{\pi}{12} + \frac{\sqrt{3}}{2} \log_e \left( \frac{2\sqrt{3}}{e} \right)$

**Sol. 2**

Given D.E. is linear D.E.

$$\text{I.F.} = e^{\int \tan x dx}$$

$$= e^{\ell n \sec x} = \sec x$$

Solution is –

$$y \sec x = \int x \sec^2 x dx$$

$$= x \tan x - \int \tan x dx$$

$$\Rightarrow y \sec x = x \tan x - \ell n \sec x + c$$

$$\text{Put } y(0) = 1$$

$$1 = 0 - 0 + c \Rightarrow c = 1$$

$$Y(x) = \frac{x \tan x}{\sec x} - \frac{\ell n \sec x}{\sec x} + \frac{1}{\sec x}$$

$$y\left(\frac{\pi}{6}\right) = \frac{\left(\frac{\pi}{6}\right)\left(\frac{1}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} - \frac{\ell n\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{\sqrt{3}}\right)} + \frac{\sqrt{3}}{2}$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2} \ell n\left(\frac{2}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2} \ell ne$$

$$= \frac{\pi}{12} - \frac{\sqrt{3}}{2} \ell n\left(\frac{2}{e\sqrt{3}}\right)$$

62. Let  $R$  be a relation on  $\mathbb{R}$ , given by

$$R = \{(a, b): 3a - 3b + \sqrt{7} \text{ is an irrational number} \}.$$

Then  $R$  is

- (1) an equivalence relation  
 (2) reflexive and symmetric but not transitive  
 (3) reflexive but neither symmetric nor transitive  
 (4) reflexive and transitive but not symmetric

**Sol. 3**

$$(a, a) \in R \Rightarrow 3a - 3a + \sqrt{7}$$

$$= \sqrt{7} \text{ (irrational)}$$

$\Rightarrow R$  is reflexive

$$\text{Let } a = \frac{2\sqrt{7}}{3} \text{ and } b = \frac{\sqrt{7}}{3}$$

$$(a, b) \in R \Rightarrow 2\sqrt{7} - \sqrt{7} + \sqrt{7}$$

$= 2\sqrt{7}$  (irrational)  
 $(b, a) \in R \Rightarrow \sqrt{7} - 2\sqrt{7} + \sqrt{7}$   
 $= 0$  (rational)  
 $\Rightarrow R$  is not symmetric  
 Let  $a = \frac{2\sqrt{7}}{3}$ ,  $b = \frac{\sqrt{7}}{3}$ ,  $c = \frac{3\sqrt{7}}{3}$   
 $(a; b) \in R \Rightarrow 2\sqrt{7}$  (irrational)  
 $(b; c) \in R \Rightarrow \sqrt{7}$  (irrational)  
 $(a, c) \in R \Rightarrow 2\sqrt{7} - 3\sqrt{7} + \sqrt{7}$   
 $= 0$  (rational)  
 $R$  is not transitive  
 $\Rightarrow R$  is reflexive but neither symmetric nor transitive

63. For a triangle  $ABC$ , the value of  $\cos 2A + \cos 2B + \cos 2C$  is least. If its inradius is 3 and incentre is  $M$ , then which of the following is NOT correct?

- (1) perimeter of  $\triangle ABC$  is  $18\sqrt{3}$
- (2)  $\sin 2A + \sin 2B + \sin 2C = \sin A + \sin B + \sin C$
- (3)  $\vec{MA} \cdot \vec{MB} = -18$
- (4) area of  $\triangle ABC$  is  $\frac{27\sqrt{3}}{2}$

**Sol. 4**

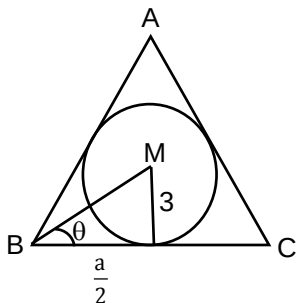
$$\begin{aligned}
 \text{Let } P &= \cos 2A + \cos 2B + \cos 2C \\
 &= 2\cos(A+B)\cos(A-B) + 2\cos^2 C - 1 \\
 &= 2\cos(\pi - C)\cos(A-B) + 2\cos^2 C - 1 \\
 &= -2\cos C [\cos(A-B) + \cos(A+B)] - 1 \\
 &= -1 - 4\cos A \cos B \cos C
 \end{aligned}$$

for  $P$  to be minimum

$\cos A \cos B \cos C$  must be maximum

$\Rightarrow \triangle ABC$  is equilateral triangle.

Let side length of triangle is  $a$



$$\tan \theta = \frac{3}{a/2}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{6}{a} \Rightarrow a = 6\sqrt{3}$$

$$\text{area of triangle} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} (108) = 27\sqrt{3}$$

64. Let  $S$  be the set of all solutions of the equation  $\cos^{-1}(2x) - 2\cos^{-1}(\sqrt{1-x^2}) = \pi$ ,  $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ . Then  $\sum_{x \in S} 2\sin^{-1}(x^2 - 1)$  is equal to
- (1)  $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$  (2)  $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$   
 (3)  $\frac{-2\pi}{3}$  (4) 0

**Sol. Bonus**

$$\cos^{-1}(2x) = \pi + 2\cos^{-1}\sqrt{1-x^2}$$

$$\text{Since } \cos^{-1}(2x) \in [0, \pi]$$

$$\text{R.H.S.} \geq \pi$$

$$\pi + 2\cos^{-1}\sqrt{1-x^2} = \pi$$

$$\Rightarrow \cos^{-1}\sqrt{1-x^2} = 0$$

$$\Rightarrow \sqrt{1-x^2} = 1$$

$$\Rightarrow x = 0$$

$$\text{but at } x = 0$$

$$\cos^{-1}(2x) = \cos^{-1}(0) = \frac{\pi}{2}$$

no solution possible for given equation.

$$x \in \phi$$

65. Let  $S$  denote the set of all real values of  $\lambda$  such that the system of equations

$$\lambda x + y + z = 1$$

$$x + \lambda y + z = 1$$

$$x + y + \lambda z = 1$$

is inconsistent, then  $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$  is equal to

- (1) 4 (2) 12 (3) 6 (4) 2

**Sol. 3**

Given system of equation is inconsistent

$$\Rightarrow \Delta = 0$$

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 3\lambda + 2 = 0$$

$$\Rightarrow (\lambda - 1)^2 (\lambda + 2) = 0$$

$$\Rightarrow \lambda = 1, -2$$

But for  $\lambda = 1$  all planes are same

Then  $\lambda = -2$

$$\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|) = 4 + 2 = 6$$

66. In a binomial distribution  $B(n, p)$ , the sum and the product of the mean and the variance are 5 and 6 respectively, then  $6(n+p-q)$  is equal to

(1) 52 (2) 50 (3) 51 (4) 53

**Sol. 1**

Given

$$np + npq = 5$$

$$\Rightarrow np(1 + q) = 5 \quad \dots(i)$$

$$\text{and } (np)(npq) = 6$$

$$\Rightarrow n^2 p^2 q = 6 \quad \dots(ii)$$

$$(i)^2 \div (ii)$$

$$\frac{(1+q)^2}{9} = \frac{25}{6}$$

$$\Rightarrow 6q^2 - 13q + 6 = 0$$

$$\Rightarrow q = \frac{2}{3}, \frac{3}{2} \text{ (reject)}$$

$$p = 1 - \frac{2}{3} = \frac{1}{3}$$

$$\frac{n}{3} \left( 1 + \frac{2}{3} \right) = 5$$

$$\Rightarrow n = 9$$

$$6(n + p - q) = 52$$

67. The combined equation of the two lines  $ax + by + c = 0$  and  $a'x + b'y + c' = 0$  can be written as  $(ax + by + c)(a'x + b'y + c') = 0$ .

The equation of the angle bisectors of the lines represented by the equation

$$2x^2 + xy - 3y^2 = 0 \text{ is}$$

$$(1) x^2 - y^2 - 10xy = 0$$

$$(2) x^2 - y^2 + 10xy = 0$$

$$(3) 3x^2 + 5xy + 2y^2 = 0$$

$$(4) 3x^2 + xy - 2y^2 = 0$$

**Sol. 1**

For pair of st. liens in form

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

equation of angle bisector is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$$\text{for } 2x^2 + xy - 3y^2 = 0$$

$$a = 2, b = -3, h = \frac{1}{2}$$

equation of angle bisector is

$$\frac{x^2 - y^2}{5} = \frac{xy}{1/2}$$

$$\Rightarrow x^2 - y^2 - 10xy = 0$$

68. The area enclosed by the closed curve  $C$  given by the differential equation  $\frac{dy}{dx} + \frac{x+a}{y-2} = 0, y(1) = 0$  is  $4\pi$ .

Let  $P$  and  $Q$  be the points of intersection of the curve  $C$  and the  $y$ -axis. If normals at  $P$  and  $Q$  on the curve  $C$  intersect  $x$ -axis at points  $R$  and  $S$  respectively, then the length of the line segment  $RS$  is

- (1) 2 (2)  $\frac{4\sqrt{3}}{3}$  (3)  $2\sqrt{3}$  (4)  $\frac{2\sqrt{3}}{3}$

Sol. 2

$$\frac{dy}{dx} + \frac{x+a}{y-2} = 0, y(1) = 0$$

$$\frac{dy}{dx} = \frac{-(x+a)}{y-2}$$

$$\int (y-2)dy = -\int (x+a)dx$$

$$\frac{y^2}{2} - 2y = -\left[\frac{x^2}{2} + ax\right] + \lambda$$

$$y(1) = 0$$

$$x = 1 \Rightarrow y = 0$$

$$0 - 0 = -\left[\frac{1}{2} + a\right] + \lambda$$

$$\frac{y^2}{2} - 2y = -\left[\frac{x^2}{2} + ax\right] + \frac{1}{2} + a$$

$$\frac{x^2 + y^2}{2} = 2y - ax + \frac{1}{2} + a$$

$$x^2 + y^2 + 2ax - 4y - 1 - 2a = 0$$

$$\text{Area} = 4\pi$$

$$\pi r^2 = 4\pi$$

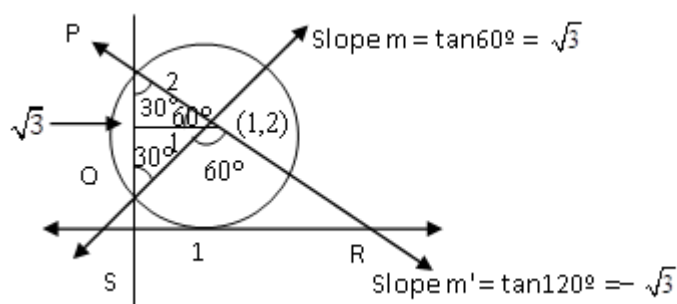
$$r^2 = 4$$

$$\alpha^2 + 4 + 1 + 2\alpha = 4$$

$$\alpha^2 + 2\alpha + 1 = 0$$

$$(\alpha + 1)^2 = 0 \Rightarrow [\alpha = -1]$$

$$x^2 + y^2 - 2x - 4y + 1 = 0$$



$$y - 2 = \sqrt{3}(x - 1)$$

$$y = 0$$

$$y - 2 = -\sqrt{3}(x - 1)$$

$$y = 0$$

$$\begin{aligned}\frac{-2}{\sqrt{3}} &= x-1 & 1+\frac{2}{\sqrt{3}} &= x \\ 1-\frac{2}{\sqrt{3}} &= x & R\left(1+\frac{2}{\sqrt{3}}, 0\right) \\ S\left(1-\frac{2}{\sqrt{3}}, 0\right) \\ RS &= \left(1+\frac{2}{\sqrt{3}}\right) - \left(1-\frac{2}{\sqrt{3}}\right) = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}\end{aligned}$$

69. The value of  $\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \dots + \frac{1}{49!2!} + \frac{1}{51!1!}$  is :

(1)  $\frac{2^{50}}{51!}$                       (2)  $\frac{2^{51}}{50!}$                       (3)  $\frac{2^{50}}{50!}$                       (4)  $\frac{2^{51}}{51!}$

Sol. 1

$$\begin{aligned}S &= \frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \dots + \frac{1}{49!2!} + \frac{1}{51!1!} \\ &= \frac{1}{51!1!} \left( \frac{51!}{1!50!} + \frac{51!}{3!48!} + \frac{51!}{5!46!} + \dots + \frac{51!}{49!2!} + \frac{51!}{51!0!} \right) \\ &= \frac{1}{51!1!} \left( {}^{51}C_{50} + {}^{51}C_{48} + {}^{51}C_{46} + \dots + {}^{51}C_2 + {}^{51}C_0 \right) \\ \therefore {}^nC_0 + {}^nC_2 + {}^nC_4 + \dots &= 2^{n-1} \\ S &= \frac{2^{50}}{51!}\end{aligned}$$

70. The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5 then the sum of cubes of the remaining two observations is

(1) 1216                      (2) 1072                      (3) 1456                      (4) 1792

Sol. 2

Let remaining two observations are a and b

$$\begin{aligned}5 &= \frac{1+3+5+a+b}{5} \\ \Rightarrow a+b &= 16 \dots (i) \\ 8 &= \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25 \\ \Rightarrow a^2+b^2 &= 130 \\ a^2+b^2 &= 130 \dots (ii) \\ (a+b)^2 &= a^2+b^2+2ab \\ \Rightarrow 256 &= 130+2ab \\ ab &= 63 \\ a^3+b^3 &= (a+b)^3 - 3ab(a+b) \\ &= (16)^3 - 3(63)(16) \\ &= 4096 - 3024 \\ \Rightarrow a^3+b^3 &= 1072\end{aligned}$$



71. The sum to 10 terms of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots \text{ is}$$

(1)  $\frac{55}{111}$

(2)  $\frac{56}{111}$

(3)  $\frac{58}{111}$

(4)  $\frac{59}{111}$

Sol. 1

$$T_n = \frac{n}{1+n^2+n^4}$$

$$= \frac{n}{(n^2-n+1)(n^2+n+1)}$$

$$= \frac{1}{2} \left[ \frac{(n^2+n+1) - (n^2-n+1)}{(n^2-n+1)(n^2+n+1)} \right]$$

$$\Rightarrow T_n = \frac{1}{2} \left[ \frac{1}{(n^2-n+1)} - \frac{1}{(n^2+n+1)} \right]$$

$$S_n = \sum_{n=1}^{10} T_n$$

$$= \frac{1}{2} \sum \left( \frac{1}{n^2-n+1} - \frac{1}{n^2+n+1} \right)$$

$$= \frac{1}{2} \left[ \left( \frac{1}{1} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{7} \right) + \left( \frac{1}{7} - \frac{1}{13} \right) \right.$$

$$\dots + \left. \left( \frac{1}{91} - \frac{1}{111} \right) \right]$$

$$= \frac{1}{2} \left[ 1 - \frac{1}{111} \right] = \frac{55}{111}$$

72. The shortest distance between the lines

$$\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3} \text{ and } \frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5} \text{ is}$$

(1)  $5\sqrt{3}$

(2)  $7\sqrt{3}$

(3)  $6\sqrt{3}$

(4)  $4\sqrt{3}$

Sol. 3

$$L_1 : \frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$$

$$\vec{a}_1 = 5\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\vec{r}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$L_2 : \frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$$

$$\vec{a}_2 = -3\hat{i} - 5\hat{j} + \hat{k}$$

$$\vec{r}_2 = \hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}$$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\text{Shortest distance (d)} = \frac{\left| (\vec{r}_1 \times \vec{r}_2) \cdot (\vec{a}_1 - \vec{a}_2) \right|}{\left| \vec{r}_1 \times \vec{r}_2 \right|}$$

$$= \frac{36}{2\sqrt{3}} = 6\sqrt{3}$$

73.  $\lim_{n \rightarrow \infty} \left[ \frac{1}{1+n} + \frac{1}{2+n} + \frac{1}{3+n} + \dots + \frac{1}{2n} \right]$  is equal to  
 (1)  $\log_e 2$  (2)  $\log_e \left( \frac{3}{2} \right)$  (3)  $\log_e \left( \frac{2}{3} \right)$  (4) 0

**Sol. 1**

$$\lim_{n \rightarrow \infty} \left[ \frac{1}{1+n} + \frac{1}{2+n} + \frac{1}{3+n} + \dots + \frac{1}{2n} \right]$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{r+n}$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n} \left( \frac{1}{\frac{r}{n} + 1} \right)$$

$$= \int_0^1 \frac{dx}{x+1}$$

$$= \log_e(1+x) \Big|_0^1$$

$$= \log_e^2$$

74. Let the image of the point  $P(2, -1, 3)$  in the plane  $x + 2y - z = 0$  be  $Q$ . Then the distance of the plane  $3x + 2y + z + 29 = 0$  from the point  $Q$  is  
 (1)  $\frac{24\sqrt{2}}{7}$  (2)  $2\sqrt{14}$  (3)  $3\sqrt{14}$  (4)  $\frac{22\sqrt{2}}{7}$

**Sol. 3**

let  $Q(\alpha, \beta, \gamma)$  is image of  $P(2, -1, 3)$  in the plane  $x + 2y - z = 0$

$$\frac{\alpha - 2}{1} = \frac{\beta + 1}{2} = \frac{\gamma - 3}{-1} = \frac{-2(2 - 2 - 3)}{1^2 + 2^2 + (-1)^2} = 1$$

$$\alpha = 3, \beta = 1, \gamma = 2$$

Distance of  $Q(3, 1, 2)$  from

$$3x + 2y + z + 29 = 0$$

$$D = \frac{|3(3) + 2(1) + 2 + 29|}{\sqrt{3^2 + 2^2 + 1^2}}$$

$$= \frac{42}{\sqrt{14}} = 3\sqrt{14}$$

75. Let  $f(x) = 2x + \tan^{-1} x$  and  $g(x) = \log_e(\sqrt{1+x^2} + x), x \in [0, 3]$ .

Then

- (1)  $\min f'(x) = 1 + \max g'(x)$
- (2)  $\max f(x) > \max g(x)$
- (3) there exist  $0 < x_1 < x_2 < 3$  such that  $f(x) < g(x), \forall x \in (x_1, x_2)$
- (4) there exists  $\hat{x} \in [0, 3]$  such that  $f'(\hat{x}) < g'(\hat{x})$

**Sol.** 2

$$f'(x) = 2 + \frac{1}{1+x^2} > 0 \quad \text{for } x \in [0, 3]$$

$$f(x) \uparrow \text{ for } x \in [0, 3]$$

$$f(0) = 0, f(3) = 6 + \tan^{-1}(3)$$

$$g'(x) = \frac{\frac{x}{\sqrt{x^2+1}} + 1}{x + \sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}} > 0 \quad \text{for } x \in [0, 3]$$

$$g(x) \uparrow \text{ for } x \in [0, 3]$$

$$g(0) = 0, g(3) = \log_e(\sqrt{10} + 3)$$

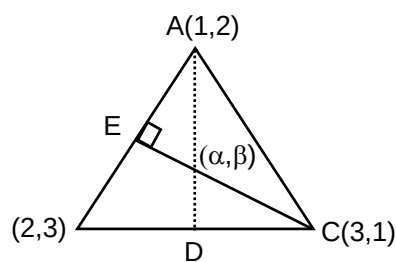
$$\max f(x) > \max g(x)$$

Option (2) correct

76. If the orthocentre of the triangle, whose vertices are (1,2) (2,3) and (3,1) is  $(\alpha, \beta)$ , then the quadratic equation whose roots are  $\alpha + 4\beta$  and  $4\alpha + \beta$ , is

- (1)  $x^2 - 20x + 99 = 0$
- (2)  $x^2 - 19x + 90 = 0$
- (3)  $x^2 - 22x + 120 = 0$
- (4)  $x^2 - 18x + 80 = 0$

**Sol.** 1



$$\text{equation of AD : } x - 2y + 3 = 0$$

$$\text{equation of CE : } x + y - 4 = 0$$

$$\text{orthocenter } (\alpha, \beta) \text{ is } \left( \frac{5}{3}, \frac{7}{3} \right)$$

$$\alpha + 4\beta = 11 \quad \text{and} \quad 4\alpha + \beta = 9$$

Quadratic equation is

$$x^2 - (11 + 9)x + (11 \times 9) = 0$$

$$\Rightarrow x^2 - 20x + 99 = 0$$

77. Let  $S = \{x: x \in \mathbb{R} \text{ and } (\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10\}$

Then  $n(S)$  is equal to

(1) 4

(2) 0

(3) 6

(4) 2

**Sol.** 1

$$(\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10$$

$$\Rightarrow (\sqrt{3} + \sqrt{2})^{x^2-4} + \frac{1}{(\sqrt{3} + \sqrt{2})^{x^2-4}} = 10$$

$$\text{Let } (\sqrt{3} + \sqrt{2})^{x^2-4} = t$$

$$t + \frac{1}{t} = 10$$

$$\Rightarrow t^2 - 10t + 1 = 0$$

$$t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$$

$$\text{If } t = 5 + 2\sqrt{6}$$

$$(\sqrt{3} + \sqrt{2})^{x^2-4} = (\sqrt{3} + \sqrt{2})^2$$

$$\Rightarrow x^2 - 4 = 2$$

$$\Rightarrow x = \pm\sqrt{6}$$

$$S = \{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$$

$$n(s) = 4$$

$$\text{If } t = 5 - 2\sqrt{6}$$

$$(\sqrt{3} + \sqrt{2})^{x^2-4} = (\sqrt{3} + \sqrt{2})^{-2}$$

$$\Rightarrow x^2 - 4 = -2$$

$$\Rightarrow x = \pm\sqrt{2}$$

78. If the center and radius of the circle  $\left| \frac{z-2}{z-3} \right| = 2$  are respectively  $(\alpha, \beta)$  and  $\gamma$ .

then  $3(\alpha + \beta + \gamma)$  is equal to

(1) 11

(2) 12

(3) 9

(4) 10

**Sol.** 2

$$\text{Put } z = x + iy$$

$$\frac{|(x-2)+iy|}{|(x-3)+iy|} = 2$$

$$\Rightarrow (x-2)^2 + y^2 = 4((x-3)^2 + y^2)$$

$$\Rightarrow x^2 - 4x + 4 + y^2 = 4x^2 - 24x + 36 + 4y^2$$

$$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$$

$$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$$

$$\text{Center } (\alpha, \beta) = \left( \frac{10}{3}, 0 \right)$$

$$\text{Radius } (\gamma) = \sqrt{\left( -\frac{10}{3} \right)^2 - \frac{32}{3}} = \frac{2}{3}$$

$$3\left( \frac{10}{3} + 0 + \frac{2}{3} \right) = 12$$

79. Let  $f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1 + \cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1 + \sin 2x \end{vmatrix}$ ,  $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ . If  $\alpha$  and  $\beta$  respectively are the maximum and the minimum values of  $f$ , then
- (1)  $\alpha^2 + \beta^2 = \frac{9}{2}$       (2)  $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$       (3)  $\alpha^2 - \beta^2 = 4\sqrt{3}$       (4)  $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$

**Sol.** 2

$$f(x) = \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1 + \cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1 + \sin 2x \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$= \begin{vmatrix} 1 + \sin^2 x & \cos^2 x & \sin 2x \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= (1 + \sin^2 x) - \cos^2 x(-1) + \sin 2x$$

$$f(x) = 2 + \sin 2x$$

$$2x \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right] \Rightarrow \frac{\sqrt{3}}{2} \leq \sin 2x \leq 1$$

$$\alpha = 2 + 1 = 3$$

$$\beta = 2 + \frac{\sqrt{3}}{2}$$

$$\beta^2 - 2\sqrt{\alpha} = \left(2 + \frac{\sqrt{3}}{2}\right)^2 - 2\sqrt{3}$$

$$= 4 + \frac{3}{4} + 2\sqrt{3} - 2\sqrt{3}$$

$$= \frac{19}{4}$$

80. The negation of the expression  $q \vee ((\sim q) \wedge p)$  is equivalent to

(1)  $(\sim p) \vee (\sim q)$       (2)  $p \wedge (\sim q)$       (3)  $(\sim p) \vee q$       (4)  $(\sim p) \wedge (\sim q)$

**Sol.** 4

$$\sim (q \vee (\sim q) \wedge p)$$

$$= \sim q \wedge (q \vee \sim p)$$

$$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$$

$$= F \vee (\sim q \wedge \sim p) = (\sim q) \wedge (\sim p)$$

### Section B

81. Let  $\vec{v} = \alpha \hat{i} + 2\hat{j} - 3\hat{k}$ ,  $\vec{w} = 2\alpha \hat{i} + \hat{j} - \hat{k}$  and  $\vec{u}$  be a vector such that  $|\vec{u}| = \alpha > 0$ . If the minimum value of the scalar triple product  $[\vec{u}\vec{v}\vec{w}]$  is  $-\alpha\sqrt{3401}$ , and  $|\vec{u} \cdot \hat{i}|^2 = \frac{m}{n}$  where  $m$  and  $n$  are coprime natural numbers, then  $m + n$  is equal to

**Sol. 3501**

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

$$= \hat{i} - 5\alpha \hat{j} - 3\alpha \hat{k}$$

$$[\vec{u} \vec{v} \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$$

$$= |\vec{u}| |\vec{v} \times \vec{w}| \cos \theta$$

$$\text{since } [\vec{u} \vec{v} \vec{w}] \text{ is Least} \Rightarrow \cos \theta = -1$$

$$[\vec{u} \vec{v} \vec{w}] = (|\vec{u}| \sqrt{1 + 25\alpha^2 + 9\alpha^2})(-1)$$

$$\Rightarrow -\alpha \sqrt{1 + 34\alpha^2} = -\alpha \sqrt{3401}$$

$$\Rightarrow \alpha^2 = 100$$

$$\Rightarrow \alpha = 10 \quad \{ \because \alpha > 0 \}$$

$$\vec{u} \text{ is parallel to } \vec{v} \times \vec{w}$$

$$\vec{u} = \lambda(\vec{v} \times \vec{w})$$

$$\vec{u} = \lambda(\hat{i} - 50\hat{j} - 30\hat{k})$$

$$|\vec{u}| = 10$$

$$|\lambda| \sqrt{3401} = 10$$

$$|\lambda| = \frac{10}{\sqrt{3401}} \quad \vec{u} = \pm \frac{10}{\sqrt{3401}}(\hat{i} - 50\hat{j} - 30\hat{k})$$

$$|\vec{u} \cdot \hat{i}|^2 = \frac{100}{3401} = \frac{m}{n}$$

$$m + n = 100 + 3401 = 3501$$

**82.** The number of words, with or without meaning, that can be formed using all the letters of the word ASSASSINATION so that the vowels occur together, is

**Sol. 50400**

$$A - 3, I - 2, S - 4, N - 2, O - 1, T - 1$$

As vowels are together

$$\text{Total words formed} = \left( \frac{8!}{4!2!} \right) \left( \frac{6!}{3!2!} \right)$$

$$= \left( \frac{8 \times 7 \times 6 \times 5}{2} \right) \left( \frac{6 \times 5 \times 4}{2} \right) = 50400$$

**83.** The remainder, when  $19^{200} + 23^{200}$  is divided by 49, is

**Sol. 29**

$$19^{200} + 23^{200} \quad a^n + b^n$$

$$19^3 = 6859 = 140 \times 49 - 1$$

$$= 49\lambda - 1$$

$$(19^3)^{66} = (49\lambda - 1)^{66}$$

So, Remainder of  $19^{198}$  divided by 49

$$\text{is } (-1)^{66} = 1$$

$19^2 = 361$  gives remainder 18  
 So,  $19^{200}$  gives remainder 18  
 $23^2$  gives remainder 39  
 $(23)^3$  gives remainder 15  
 $(23)^4$  gives remainder 2  
 $((23)^4)^6$  gives remainder  $(2)^6 = 64$   
 & 64 gives remainder 15  
 $(23)^{24} \longrightarrow 15$   
 $(23)^{25} \longrightarrow 2$   
 $((23)^{25})^8 \longrightarrow (2)^8 = 256 \longrightarrow 11$   
 So, Total remainder =  $18 + 11 = 29$

**84.** The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7, is  
**Sol.** 514

3 digit numbers divisible by either 2 or 3  
 $P = n(\text{divisible by 2}) + n(\text{divisible by 3}) - n(\text{divisible by 6})$   
 $P = 450 + 300 - 150$   
 $P = 600$   
 $Q = n(\text{divisible by 14}) + n(\text{divisible by 21}) - n(\text{divisible by 42})$   
 $= 64 + 43 - 21 = 86$   
 3 digit number divisible by either 2 or 3  
 But not divisible by 7 so  $P - Q = 600 - 86 = 514$

**85.** Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f'(x) + f(x) = \int_0^2 f(t) dt$ .  
 If  $f(0) = e^{-2}$ , then  $2f(0) - f(2)$  is equal to

**Sol.** 1

Let  $\int_0^2 f(t) dt = \lambda$   
 $f'(x) + f(x) = \lambda$   
 is linear Differential equation  
 I.f. =  $e^{\int dx} = e^x$   
 $f(x).e^x = \int e^x \lambda dx$   
 $\Rightarrow f(x).e^x = \lambda e^x + C$   
 $\Rightarrow f(x) = \lambda + Ce^{-x}$   
 put  $f(0) = e^{-2}$   
 $e^{-2} = \lambda + C \Rightarrow C = e^{-2} - \lambda$   
 $f(x) = \lambda + (e^{-2} - \lambda) e^{-x}$   
 $\lambda = \int_0^2 f(t) dt$   
 $= \int_0^2 (\lambda + (e^{-2} - \lambda) e^{-t}) dt$   
 $\Rightarrow \lambda = \lambda + \lambda e^{-2} - e^{-4} + e^{-2}$   
 $\Rightarrow \lambda = e^{-2} - 1$   
 $f(x) = e^{-2} - 1 + e^{-x}$

$$f(0) = e^{-2}$$

$$f(2) = 2e^{-2} - 1$$

$$2f(0) - f(2) = 1$$

86. If  $f(x) = x^2 + g'(1)x + g''(2)$  and  $g(x) = f(1)x^2 + xf'(x) + f''(x)$ , then the value of  $f(4) - g(4)$  is equal to

**Sol. 14**

$$\text{let } g'(1) = A$$

$$g''(2) = B$$

$$f(x) = x^2 + Ax + B$$

$$f(1) = A + B + 1$$

$$f'(x) = 2x + A$$

$$f''(x) = 2$$

$$g(x) = (A + B + 1)x^2 + x(2x + A) + 2$$

$$\Rightarrow g(x) = x^2(A + B + 2) + Ax + 2$$

$$g'(x) = 2x(A + B + 2) + A$$

$$g'(1) = A$$

$$\Rightarrow 2(A + B + 2) + A = A$$

$$A + B = -2 \quad \dots(i)$$

$$g''(x) = 2(A + B + 2)$$

$$g''(2) = B$$

$$\Rightarrow 2(A + B + 2) = B$$

$$\Rightarrow 2A + B = -4 \quad \dots(ii)$$

From (i) and (ii)

$$A = -2 \text{ and } B = 0$$

$$f(x) = x^2 - 2x$$

$$f(4) = 16 - 8 = 8$$

$$g(x) = -2x + 2$$

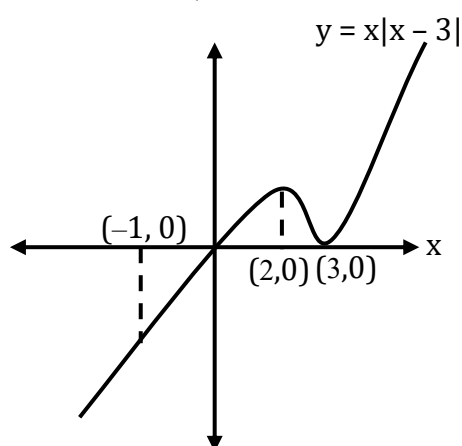
$$g(4) = -8 + 2 = -6$$

$$f(4) - g(4) = 8 - (-6) = 14$$

87. Let  $A$  be the area bounded by the curve  $y = x|x - 3|$ , the  $x$ -axis and the ordinates  $x = -1$  and  $x = 2$ . Then  $12A$  is equal to

**Sol. 62**

$$y = x|x - 3| = \begin{cases} x(x-3); & x \geq 3 \\ x(3-x); & x < 3 \end{cases}$$





$$\begin{aligned}
A &= -\int_{-1}^0 x(3-x)dx + \int_0^2 x(3-x)dx \\
&= \int_{-1}^0 (x^2 - 3x)dx + \int_0^2 (3x - x^2)dx \\
&= \left[ \frac{x^3}{3} - \frac{3x^2}{2} \right]_{-1}^0 + \left[ \frac{3x^2}{2} - \frac{x^3}{3} \right]_0^2 \\
A &= 0 - \left( \frac{-1}{3} - \frac{3}{2} \right) + 6 - \frac{8}{3} = \frac{31}{6} \\
A &= 12 \left( \frac{31}{6} \right) = 62
\end{aligned}$$

- 88.** If  $\int_0^1 (x^{21} + x^{14} + x^7)(2x^{14} + 3x^7 + 6)^{1/7} dx = \frac{1}{\ell} (11)^{m/n}$  where  $l, m, n \in \mathbb{N}$ ,  $m$  and  $n$  are coprime then  $l + m + n$  is equal to

**Sol. 63**

$$\begin{aligned}
I &= \int_0^1 (x^{21} + x^{14} + x^7) (2x^{14} + 3x^7 + 6)^{\frac{1}{7}} dx \\
&= \int_0^1 (x^{20} + x^{13} + x^6) (2x^{14} + 3x^7 + 6)^{\frac{1}{7}} dx \\
\text{Put } 2x^{14} + 3x^7 + 6 &= t \\
\Rightarrow 42(x^{20} + x^{13} + x^6)dx &= dt \\
\Rightarrow (x^{20} + x^{13} + x^6)dx &= \frac{dt}{42}
\end{aligned}$$

$$\begin{aligned}
I &= \int_0^{11} \frac{t^{\frac{1}{7}}}{42} dt \\
&= \frac{1}{42} \left[ \frac{t^{\frac{8}{7}}}{\frac{8}{7}} \right]_0^{11} \\
&= \left( \frac{7}{8} \right) \left( \frac{1}{42} \right) (11)^{8/7} \\
&= \frac{1}{48} (11)^{8/7} = \frac{1}{\ell} (11)^{m/n}
\end{aligned}$$

$$\ell + m + n = 48 + 8 + 7 = 63$$

- 89.** Let  $a_1 = 8, a_2, a_3, \dots, a_n$  be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is

**Sol. 754**

$$a_1 = 8$$

$d$  = common difference

$$\frac{4}{2} [16 + 3d] = 50$$

$$\Rightarrow d = 3$$

$$\frac{4}{2} [2a_n + 3(-d)] = 170$$

$$\Rightarrow 2(a_1 + (n-1)d) - 3d = 85$$

$$\Rightarrow 16 + 6(n-1) - 9 = 85$$

$$n - 1 = 13$$

$$n = 14$$

Product of middle two terms =  $T_7 \times T_8$

$$= (a_1 + 6d)(a_1 + 7d)$$

$$= (8 + 18)(8 + 21)$$

$$= (26)(29) = 754$$

90.  $A(2,6,2), B(-4,0,\lambda), C(2,3,-1)$  and  $D(4,5,0), |\lambda| \leq 5$  are the vertices of a quadrilateral  $ABCD$ . If its area is 18 square units, then  $5 - 6\lambda$  is equal to

Sol. 11

$$\overrightarrow{AD} = 2\hat{i} - \hat{j} - 2\hat{k}$$

$$\overrightarrow{AC} = -3\hat{j} - 3\hat{k}$$

$$\overrightarrow{AD} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & -2 \\ 0 & -3 & -3 \end{vmatrix}$$

$$= -3\hat{i} + 6\hat{j} - 6\hat{k}$$

$$\text{Area}(\triangle ADC) = \frac{1}{2} |\overrightarrow{AD} \times \overrightarrow{AC}|$$

$$= \frac{1}{2} \sqrt{9+36+36} = \frac{9}{2}$$

$$\overrightarrow{AB} = -6\hat{i} - 6\hat{j} + (\lambda - 2)\hat{k}$$

$$\overrightarrow{AC} = -3\hat{j} - 3\hat{k}$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -6 & \lambda - 2 \\ 0 & -3 & -3 \end{vmatrix}$$

$$= (12 + 3\lambda)\hat{i} - 18\hat{j} + 18\hat{k}$$

$$\text{area}(\triangle ABC) = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$= \frac{3}{2} \sqrt{(4 + \lambda)^2 + 36 + 36}$$

$$\text{Area}(\triangle ABCD) = \text{ar}(\triangle ADC) + \text{ar}(\triangle ABC)$$

$$\Rightarrow 18 = \frac{9}{2} + \frac{3}{2} \sqrt{(4 + \lambda)^2 + 72}$$

$$\Rightarrow (4 + \lambda)^2 = 9$$

$$4 + \lambda = 3 \quad \text{or} \quad 4 + \lambda = -3$$

$$\Rightarrow \lambda = -1 \quad \text{or} \quad \lambda = -7 \text{ (reject)}$$

$$5 - 6\lambda = 5 + 6 = 11$$

