SOLVED PAPER – 2019 (COMEDK)

Instructions

- There are 180 questions in all. The number of questions in each section is as given below.

Sections	No. of Questions
Section I : Physics	1–60
Section II : Chemistry	61–120
Section III : Mathematics	121–180

- All the questions are Multiple Choice Questions having four options out of which **ONLY ONE** is correct.
- Candidates will be awarded 1 mark for each correct answer. There will be no negative marking for incorrect answer.
- Time allotted to complete this paper is 3 hrs.

PHYSICS

1. A thin plano-convex lens acts like a concave mirror of focal length 0.2 m when silvered from its plane surface. The refractive index of the material of the lens is 1.5. The radius of curvature of the convex surface of the lens will be

a. 0.4 m ***b.*** 0.2 m
c. 0.1 m ***d.*** 0.75 m

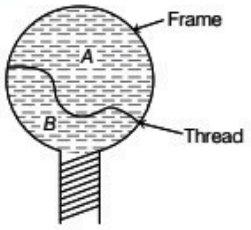
2. The physical quantity having the same dimensions as Planck's constant h is

a. Boltzmann constant
b. force
c. linear momentum
d. angular momentum

3. A balloon is rising vertically up with a velocity of 29 ms^{-1}. A stone is dropped from it and it reaches the ground in 10 s. The height of the balloon when the stone was dropped from it is ($g = 9.8\ ms^{-2}$)

a. 100 m ***b.*** 200 m
c. 400 m ***d.*** 150 m

4. A thread is tied slightly loose to a wire frame as in figure and the frame is dipped into a soap solution and taken out. The frame is completely covered with the film. When the portion A is punctured with a pin, the thread



a. becomes concave towards A
b. becomes convex towards A
c. remains in the initial position
d. Either (a) or (b) depending on the size of A w.r.t. B

5. Oxygen is 16 times heavier than hydrogen. Equal volumes of hydrogen and oxygen are mixed. The ratio of speed of sound in the mixture to that in hydrogen is

a. $\sqrt{1/8}$ ***b.*** $\sqrt{\frac{32}{17}}$ ***c.*** $\sqrt{8}$ ***d.*** $\sqrt{\frac{2}{17}}$

6. When light is incident on a diffraction grating, the zero order principal maximum will be
a. one of the component colours
b. absent
c. spectrum of the colours
d. white

7. H-polaroid is prepared by
a. stretching polyvinyl alcohol and then heated with dehydrating agent
b. stretching polyvinyl alcohol and then impregnating with iodine
c. orienting herapathite crystal in the same direction in nitrocellulose
d. by using thin tourmaline crystals

8. SI unit of permittivity is
a. $C^2m^2N^{-1}$ *b.* $C^{-1}m^2N^{-2}$
c. $C^2m^2N^2$ *d.* $C^2m^{-2}N^{-1}$

9. A spherical drop of capacitance 1μ F is broken into eight drops of equal radius. Then, the capacitance of each small drop is
a. $\frac{1}{8}\mu F$ *b.* $8\mu F$
c. $\frac{1}{2}\mu F$ *d.* $\frac{1}{4}\mu F$

10. Two equal forces (p each) act at a point inclined to each other at an angle of 120°. The magnitude of their resultant is
a. p *b.* $2p$
c. $p/2$ *d.* $p/4$

11. If two waves of the same frequency and amplitude respectively on superposition produce a resultant disturbance of the same amplitude the waves differ in phase by
a. $\pi/3$ *b.* $2\pi/3$
c. π *d.* zero

12. A man, standing between two cliffs, claps his hands and starts hearing a series of echoes at intervals of one second. If the speed of sound in air is 340 ms^{-1}, then the distance between the cliffs, is
a. 340 m *b.* 1620 m
c. 680 m *d.* 1700 m

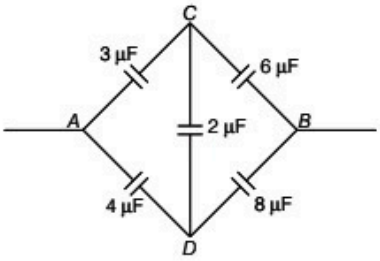
13. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is
a. 1.2 mm *b.* 1.2 cm
c. 2.4 cm *d.* 2.4 mm

14. Specific rotation of sugar solution is 0.01 SI units. 200 kgm^{-3} of impure sugar solution is taken in a polarimeter tube of length 0.25 m and an optical rotation of 0.4 rad is observed. The percentage of purity of sugar in the sample is
a. 80% *b.* 89%
c. 11% *d.* 20%

15. An electron is accelerated through a potential difference of 45.5 V. The velocity acquired by it (in ms^{-1}) is
a. 4×10^6 *b.* 4×10^4
c. 10^6 *d.* zero

16. When a body is earth connected, electrons from the earth flow into the body. This means, the body is
a. uncharged *b.* charged positively
c. charged negatively *d.* an insulator

17. Effective capacitance between A and B in the figure shown is (all capacitances are in μ F)



a. 21 μ F *b.* 23μ F
c. $\frac{3}{14}\mu F$ *d.* $\frac{14}{3}\mu F$

18. Which state of triply ionised Beryllium (Be^{+++}) has the same orbital radius as that of the ground state of hydrogen ?
a. $n = 1$ *b.* $n = 2$
c. $n = 3$ *d.* $n = 4$

19. If M is the atomic mass and A is the mass number, packing fraction is given by
a. $\frac{A}{M-A}$ *b.* $\frac{A-M}{A}$
c. $\frac{M}{M-A}$ *d.* $\frac{M-A}{A}$

20. A count rate meter shows a count of 240 per min from a given radioactive source. One hour later the meter shows a count rate of 30 per min. The half-life of the source is
a. 20 min *b.* 30 min
c. 80 min *d.* 120 min

21. The refractive index of a particular material is 1.67 for blue light, 1.65 for yellow light and 1.63 for red light. The dispersive power of the material is

a. 0.0615 ***b.*** 0.024 ***c.*** 0.031 ***d.*** 1.60

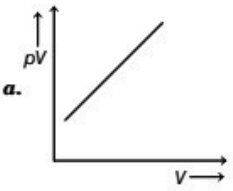
22. An ideal gas heat engine operates in a Carnot's cycle between 227°C and 127°C. It absorbs 6×10^4 J at high temperature. The amount of heat converted into work is

a. 4.8×10^4 J ***b.*** 3.5×10^4 J

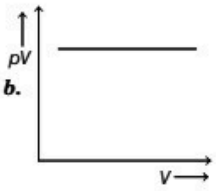
c. 1.6×10^4 J ***d.*** 1.2×10^4 J

23. Which one of the following graphs represents the behaviour of an ideal gas?

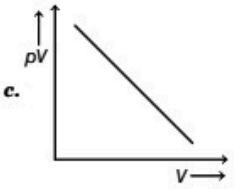
a.



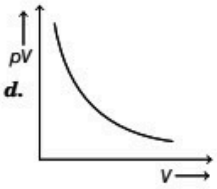
b.



c.



d.



24. Rainbow is formed due to

a. refraction

b. dispersion and total internal reflection

c. total internal reflection

d. scattering

25. A beam of parallel rays is brought to a focus by a plano-convex lens. A thin concave lens of the same focal length is joined to the first lens. The effect of this is

a. the focal point shifts away from the lens by a small distance

b. the focus remains undisturbed

c. the focus shifts to infinity

d. the focal point shifts towards the lens by a small distance

26. Two conductors of the same material have their diameters in the ratio 1 : 2 and their lengths in the ratio 2 : 1. If the temperature difference between their ends is the same, then the ratio of amounts of heat conducted per second through them will be

a. 8 : 1 ***b.*** 1 : 8 ***c.*** 4 : 1 ***d.*** 1 : 4

27. Blowing air with open mouth is an example of

a. isothermal process

b. adiabatic process

c. isobaric process

d. isochoric process

28. Sound waves in air are always longitudinal because

a. air is a mixture of several gases

b. density of air is very small

c. of the inherent characteristics of sound waves in air

d. air does not have a modulus of rigidity

29. In Young's double slit experiment, if monochromatic light used is replaced by white light, then

a. all bright fringes become white

b. all bright fringes have colours between violet and red

c. no fringes are observed

d. only central fringe is white, all other fringes are coloured

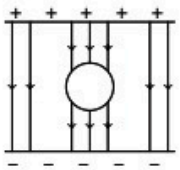
30. In a Young's double slit experiment, the separation between the two slits is 0.9 mm and the fringes are observed 1m away. If it produces the second dark fringe at a distance of 1 mm from the central fringe, the wavelength of the monochromatic source of light used is

a. 500 nm ***b.*** 600 nm

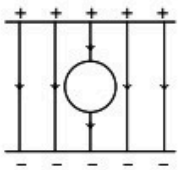
c. 450 nm ***d.*** 400 nm

31. An uncharged sphere of metal is placed inside a charged parallel plate capacitor. The lines of force will look like

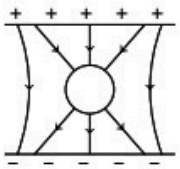
a.



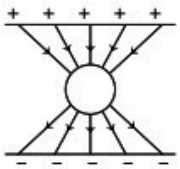
b.



c.



d.



32. A wire has a resistance of 6Ω. It is cut into two parts and both half values are connected in parallel. The new resistance is

a. 12 Ω ***b.*** 15 Ω

c. 3 Ω ***d.*** 6 Ω

33. A current flows in a conductor from East to West. The direction of the magnetic field at a point above the conductor is

a. towards North ***b.*** towards South
c. towards East ***d.*** towards West

34. A bar magnet is equivalent to

a. solenoid carrying current
b. circular coil carrying current
c. torroid carrying current
d. straight conductor carrying current

35. Excitation energy of a hydrogen like ion in its first excitation state is 40.8 eV. Energy needed to remove the electron from the ion in ground state is

a. 54.4 eV ***b.*** 13.6 eV
c. 40.8 eV ***d.*** 27.2 eV

36. Threshold wavelength for photoelectric emission from a metal surface is 5200 Å. Photoelectrons will be emitted when this surface is illuminated with monochromatic radiation from

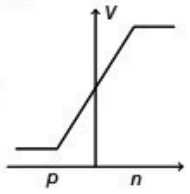
a. 50 W IR-lamp ***b.*** 10 W IR-lamp
c. 1 W IR-lamp ***d.*** 50 W UV-lamp

37. The emitter-base junction of a transistor is ..... biased, while the collector-base junction is ..... biased.

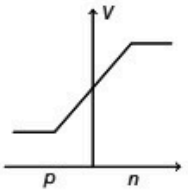
a. reverse, forward ***b.*** reverse, reverse
c. forward, forward ***d.*** forward, reverse

38. In a forward biased *p-n* junction diode, the potential barrier in the depletion region is of the form

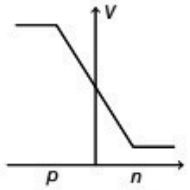
a.



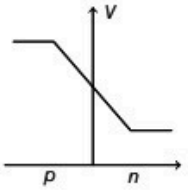
b.



c.



d.



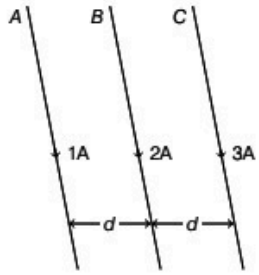
39. A cylinder of radius r and length l is placed in an uniform electric field E parallel to the axis of the cylinder. The total flux for the surface of the cylinder is given by

a. $\pi r^2 E$ ***b.*** $(\pi r^2 + \pi l^2)E$
c. Zero ***d.*** $2\pi r^2 E$

40. Two electric bulbs *A* and *B* are rated as 60 W and 100 W. They are connected in parallel to the same source. Then,

a. Both draw the same current
b. *A* draws more current than *B*
c. *B* draws more current than *A*
d. currents drawn are in the ratio of their resistances

41. Three long straight wires *A*, *B* and *C* are carrying currents as shown in figure. Then, the resultant force on *B* is directed



a. towards *A*
b. towards *C*
c. perpendicular to the plane of paper and outward
d. perpendicular to the plane of paper and inward

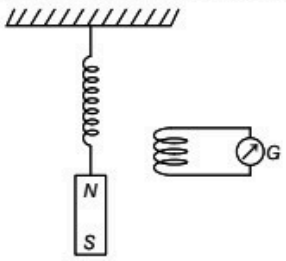
42. Curie-Weiss law is obeyed by iron at a temperature

a. below Curie temperature
b. above Curie temperature
c. at Curie temperature only
d. at all temperatures

43. The dimensional formula for inductance is

a. $[ML^2T^{-1}A^{-2}]$ ***b.*** $[ML^2T^{-2}A^{-1}]$
c. $[ML^2T^{-2}A^{-2}]$ ***d.*** $[ML^2TA^{-2}]$

44. A magnet *NS* is suspended from a spring and while it oscillates, the magnet moves in and out of the coil *C*. The coil is connected to a galvanometer *G*. Then, as the magnet oscillates,



a. *G* shows deflection to the left and right with constant amplitude
b. *G* shows deflection on one side
c. *G* shows no deflection
d. *G* shows deflection to the left and right but the amplitude steadily decreases

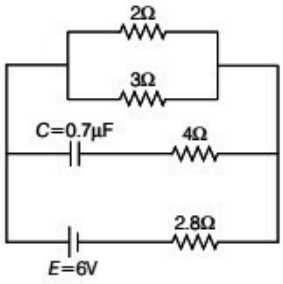
45. The maximum current that can be measured by a galvanometer of resistance 40 Ω is 10 mA. It is converted into a voltmeter that can read upto 50 V. The resistance to be connected in series with the galvanometer (in ohm) is

a. 5040 *b.* 4960 *c.* 2010 *d.* 4050

46. An unknown resistance R_1 is connected in series with a resistance of 10 Ω. This combination is connected to one gap of a meter bridge, while a resistance R_2 is connected in the other gap. The balance point is at 50 cm. Now, when the 10 Ω resistance is removed the balance point shifts to 40 cm. The value of R_1 (in ohm) is

a. 60 *b.* 40 *c.* 20 *d.* 10

47. In the circuit shown, the internal resistance of the cell is negligible. The steady state current in the 2 Ω resistor is



a. 0.9A *b.* 1.5 A
c. 0.6 A *d.* 1.2 A

48. A rectangular coil of 300 turns has an average area of 25 cm × 10 cm. The coil rotates with a speed of 50 cps in a uniform magnetic field of strength 4×10^{-2} T about an axis perpendicular to the field. The peak value of the induced emf (in volt) is

a. 3π *b.* 30 π
c. 300 π *d.* 3000 π

49. In a *L-C-R* circuit the potential difference between the terminals of the inductance is 60 V, between the terminals of the capacitor is 30 V and that between the terminals of resistance is 40 V. The supply voltage will be equal to

a. 50 V *b.* 70 V
c. 130 V *d.* 10 V

50. A vertical circular coil of radius 0.1 m and having 10 turns carries a steady current. When the plane of the coil is normal to the magnetic meridian, a neutral point is observed at the centre of the coil. If $B_H = 0.314 \times 10^{-4}$ T, then the current in the coil is

a. 2 A *b.* 1 A
c. 0.5 A *d.* 0.25 A

51. The spectrum obtained from the chromosphere of the sun at the time of total solar eclipse is

a. continuous emission spectrum
b. line absorption spectrum
c. line emission spectrum
d. band absorption spectrum

52. Heavy water is

a. water, in which soap does not lather
b. compound of heavy oxygen and heavy hydrogen
c. compound of deuterium and oxygen
d. water at 4°C

53. The nuclear reactor at Kaiga is a

a. breeder reactor
b. power reactor
c. research reactor
d. fusion reactor

54. When a body moves in a circular path, no work is done by the force since

a. there is no displacement
b. there is no net force
c. force and displacement are perpendicular to each other
d. the force is always away from the centre

55. A bullet moving with a speed of 100 ms^{-1} can just penetrate two planks of equal thickness. Then, the number of such planks penetrated by the same bullet when the speed is doubled will be

a. 4 *b.* 8
c. 6 *d.* 10

56. Two bodies of masses 1 kg and 2 kg have equal momentum. Then, the ratio of their kinetic energies is

a. 1 : 3 *b.* 1 : 1
c. 2 : 1 *d.* 3 : 1

57. The loudness and pitch of a sound note depends on

a. intensity and frequency
b. frequency and number of harmonics
c. intensity and velocity
d. frequency and velocity

58. Absorption co-efficient of an open window is

a. Zero *b.* 0.5
c. 1 *d.* 0.25

59. In Melde's experiment in the transverse mode, the frequency of the tuning fork and the frequency of the waves in the string are in the ratio
a. 1 : 1 *b.* 1 : 2 *c.* 2 : 1 *d.* 4 : 1

60. The difference between the apparent frequency of a source of sound as perceived by the observer during its approach and recession is 2% of the frequency of the source. If the speed of sound in air is 300 ms^{-1}, then the velocity of the source is
a. 6 ms^{-1} *b.* 3 ms^{-1}
c. 1.5 ms^{-1} *d.* 12 ms^{-1}

CHEMISTRY

61. Ozone in stratosphere is depleted by
a. CF_2Cl_2 *b.* C_7F_{16} *c.* $C_6H_6Cl_2$ *d.* C_6F_6

62. Which one of the following is an unsaturated fatty acid?
a. Palmitic acid *b.* Lauric acid
c. Linolenic acid *d.* Myristic acid

63. When chlorine is passed through boiling toluene, we get
a. *o*-chloro toluene
b. *p*-chlorotoluene
c. mixture of *o* and *p*-chlorotoluene
d. benzyl chloride

64. The standard temperature used in thermo chemical calculations is
a. 273 K *b.* 298 K *c.* 297 K *d.* 303 K

65. Which of the following is an intensive property?
a. Enthalpy *b.* Entropy
c. Density *d.* Mass

66. Schiff's reagent contains
a. rochelle salt *b.* resorcinol
c. rosaniline *d.* α-naphthol

67. The formula of chromyl chloride is
a. CrCl *b.* $CrCl_3$ *c.* $CrOCl_2$ *d.* CrO_2Cl_2

68. Horn silver is
a. oxide ore *b.* sulphide ore
c. halide ore *d.* carbonate ore

69. Tetrahedral structure is formed by
a. sp^2-hybridisation *b.* sp^3-hybridisation
c. dsp^3-hybridisation *d.* d^2sp^3-hybridisation

70. NO^+ ligand is
a. nitronium *b.* nitrosyl
c. nitrosonium *d.* nitro

71. Cationic complex is
a. hexa amino platinum chloride
b. potassium ferro cyanide
c. sodium argento cyanide
d. nickel carbonyl

72. $2p_x$ atomic orbital undergoes linear combination with
a. $2p_y$ orbital
b. $2p_x$ orbital
c. Both $2p_y$ and $2p_z$ orbitals
d. $2p_z$ orbital

73. In a first order reaction, molar concentration of a reactant decreases from 0.1 to 0.01 in 100 s. The rate constant of the reaction is
a. 2.3030 *b.* 0.02303
c. 0.2303 *d.* 0.002303

74. In which one of the following equilibria, pressure has no effect
a. $PCl_5 \rightleftharpoons PCl_3 + Cl_2$
b. $2NH_3 \rightleftharpoons N_2 + 3H_2$
c. $2SO_2 + O_2 \rightleftharpoons 2SO_3$
d. $N_2 + O_2 \rightleftharpoons 2NO$

75. Conductivity of a solution is not affected by
a. addition of water
b. process of heating
c. addition of acetic acid
d. addition of ethanol

76. The lowering in vapour pressure is maximum for
a. 0.1 M urea
b. 0.1 M NaCl
c. 0.1 M $MgCl_2$
d. 0.1 M $K_4[Fe(CN)_6]$

77. Bromo ethane and isopropyl chloride with metallic sodium in ether forms
a. pentane *b.* 2-methyl butane
c. 3-methyl butane *d.* 2,3-dimethyl butane

78. To dry ammonia gas the drying agent used is
a. conc. H_2SO_4 *b.* P_2O_5
c. soda lime *d.* anhydrous $CaCl_2$

79. The metal hydroxide which is soluble in excess of ammonium hydroxide is
a. $Fe(OH)_2$ *b.* $Fe(OH)_3$
c. $Cu(OH)_2$ *d.* $Al(OH)_3$

80. Potassium dichromate can be converted to potassium chromate by adding
a. KOH *b.* conc. H_2SO_4
c. NH_4OH *d.* acetic acid

81. 0.5 g of an acid is neutralised by 40 cc of 0.125 N NaOH. The equivalent mass of the acid is
a. 50 *b.* 100 *c.* 40 *d.* 80

82. 5 L of NaOH solution of pH 12 contains
a. 200 g *b.* 0.2 g
c. 20 g *d.* 2g

83. 50 cc of oxalic acid is oxidised by 25 cc of 0.20 N $KMnO_4$. The mass of oxalic acid present in 500 cc of the solution is
a. 3.15 g *b.* 31.5 g *c.* 6.3 g *d.* 63 g

84. Pure water is neutral because
a. pH = 7
b. litmus has no effect
c. it is free from dissolved salts
d. pH = 0

85. In the titration of mohr salt against $KMnO_4$, the indicator used is
a. diphenyl amine *b.* $KMnO_4$
c. phenolphthalein *d.* methyl orange

86. The relationship between half-life of a reaction and the order of reaction is
a. $t_{\frac{1}{2}} \propto \frac{1}{a^{(n+1)}}$ *b.* $t_{\frac{1}{2}} \propto \frac{1}{a^{(n+2)}}$
c. $t_{\frac{1}{2}} \propto \frac{1}{a^{n}}$ *d.* $t_{\frac{1}{2}} \propto \frac{1}{a^{(n-1)}}$

87. 6g of urea is dissolved in 90g of water. Relative lowering of vapour pressure is
a. 0.02 *b.* 0.2 *c.* 0.002 *d.* 0.04

88. 6.84g of sucrose is dissolved in 200g of water. The molality of the solution is
a. 0.2 M *b.* 0.3 M *c.* 0.1 M *d.* 0.02 M

89. When common salt is added of a saturated solution of soap is precipitated. This is based on the principle of
a. common ion effect
b. principle of solubility product
c. adsorption from solution
d. peptisation

90. Highest osmotic pressure is shown by a solution of
a. 0.1 M aluminium sulphate
b. 0.1 M potassium nitrate
c. 0.1 M magnesium chloride
d. 0.1 M barium chloride

91. 50% of first order reaction is completed in 30 min. The velocity constant of the reaction is
a. 0.231 *b.* 2.31 *c.* 0.00231 *d.* 0.0231

92. The ebullioscopic constant is the elevation in boiling point produced by
a. 1 molar solution *b.* 1 molal solution
c. 1 N solution *d.* 10% solution

93. The mass of glucose to be dissolved in 50g of water to get 0.3 molal solution is
a. 27g *b.* 0.27 g *c.* 2.7 g *d.* 5.4 g

94. 25 mL of 0.08 N Mohr salt solution is oxidised by 20 mL of $K_2Cr_2O_7$ in acid medium. The mass of Mohr's salt present in 500 cc is
a. 3.96 g *b.* 19.6 g
c. 39.6 g *d.* 39.2 g

95. A reaction is spontaneous at all temperature, when
a. ΔH is –ve and ΔS is + ve
b. ΔH is + ve and ΔS is – ve
c. Both ΔH and ΔS are – ve
d. Both ΔH and ΔS are + ve

96. The coordination number of sodium chloride is
a. 4 *b.* 8 *c.* 6 *d.* 12

97. Conjugate acid of NH_2^- is
a. NH_3 *b.* NH_4^+
c. N *d.* NH_2^+

98. Highest molar conductivity is given by
a. 0.005 M NaCl *b.* 0.1 M NaCl
c. 0.05 M NaCl *d.* 0.01 M NaCl

99. In the detection of III group basic radicals NH_4OH is added after NH_4Cl to
a. increase in the ionisation of NH_4OH
b. increase in the ionisation of salt solution
c. decrease in the ionisation of salt solution
d. decrease in the ionisation of NH_4OH

100. Just before attaining the chemical equilibrium
a. rate of forward reaction decreases and rate of backward reaction increases
b. rate of forward reaction increases and rate of backward reaction decreases
c. no change in the rates of forward and backward reactions.
d. rate of forward reaction equals the rate backward reaction.

101. Which one of the following shows highest magnetic moment?
a. Fe^{2+} *b.* Co^{2+}
c. Cr^{3+} *d.* Ni^{2+}

102. In 3rd series as we move from scandium to zinc the paramagnetism
a. increases
b. decreases
c. first increases to a maximum and then decreases
d. first decreases to a minimum and then increases

103. The number of unpaired electrons in Fe^{3+} is
a. 2 *b.* 3
c. 4 *d.* 5

104. The IUPAC name of $K_4[Fe(CN)_6]$ is
a. potassium ferri cyanide
b. potassium ferro cyanide
c. potassium hexa cyano ferrate (II)
d. potassium hexa cyano ferrate (III)

105. The adsorption of an inert gases on activated charcoal increase with
a. decrease of pressure
b. increase of temperature
c. decrease of atomic mass
d. decrease of temperature

106. Electrolysis of brine gives a mixture of
a. H_2, Na, Cl_2 *b.* Cl_2, H_2, NaOH
c. H_2, O_2, NaOH *d.* O_2, Cl_2, NaOH

107. Sucrose is a non-reducing sugar due to
a. 1-2 linkage *b.* 1-4 linkage
c. 1-5 linkage *d.* 1-6 linkage

108. Sulphur containing amino acid is
a. alanine *b.* proline
c. tyrosine *d.* cysteine

109. Lysine is
a. neutral amino acid
b. acidic amino acid
c. basic amino acid
d. heterocyclic amino acid

110. In the Molisch reagent, the substance used is
a. β-naphthol in alcohol
b. α-naphthol in alcohol
c. Resorcinal in alcohol
d. Resonance in water

111. In benzene, each carbon atom undergoes
a. *sp* hybridisation
b. sp^2 hybridisation
c. sp^3 hybridisation
d. dsp^2 hybridisation

112. When vapours of isopropyl alcohol is passed over heated copper, we get acetone. It is an example for
a. dehydration *b.* dehalogenation
c. dehydrohalogenation *d.* dehydrogenation

113. The IUPAC name of $CH_3-\underset{\displaystyle CH_3}{\overset{|}{N}}-CH_3$ is
a. trimethyl amine
b. 2-methyl ethanamine
c. N, N-dimethyl methanamine
d. trimethyl ammonia

114. When benzaldehyde is condensed with acetic anhydride in presence of fused sodium acetate, we get
a. crotonic acid *b.* cinnamic acid
c. aspartic acid *d.* salicylic acid

115. When a mixture of calcium benzoate and calcium formate is dry distilled, we get
a. formaldehyde *b.* acetaldehyde
c. benzaldehyde *d.* salicylaldehyde

116. Which one of the following is strongly basic?
a. Dimethyl amine *b.* Methyl amine
c. Ammonia *d.* Aniline

117. Which one of the following is bi-functional compound?
a. Formic acid *b.* Acetic acid
c. Benzoic acid *d.* Cinnamic acid

118. When phenol is treated with chloromethane in presence of $AlCl_3$ we get
a. *o*-cresol
b. *m*-cresol
c. *p*-cresol
d. mixture of *o* and *p*-cresol

119. In the synthesis of ammonia

$$N_2 + 3H_2 \rightleftharpoons 2NH_3$$

a. $K_p = K_C RT$ *b.* $K_p = K_C$
c. $K_p = K_C(RT)^{-2}$ *d.* $K_p = K_C(RT)^{-1}$

120. When the same amount of electricity is passed through solutions of silver nitrate and copper sulphate, 0.4 g copper is deposited. The amount of silver deposited is
a. 1.35 g *b.* 2.7 g
c. 5.1 g *d.* 5.4 g

121. A variable line $\frac{x}{a}+\frac{y}{b}=1$ is such that $a+b=4$. The locus of the mid-point of the portion of the line intercepted between the axes is

a. $x+y=4$ *b.* $x+y=8$
c. $x+y=1$ *d.* $x+y=2$

122. The point $(5,-7)$ lies outside the circle

a. $x^2+y^2-8x=0$
b. $x^2+y^2-5x+7y=0$
c. $x^2+y^2-5x+7y-1=0$
d. $x^2+y^2-8x+7y-2=0$

123. If the circle $x^2+y^2=9$ and $x^2+y^2+2\alpha x+2y+1=0$ touch each other internally, then $\alpha=$

a. $\pm\frac{4}{3}$ *b.* 1
c. $\frac{4}{3}$ *d.* $\frac{-4}{3}$

124. The locus of the mid-points of the line joining the focus and point on the parabola $y^2=4ax$ is a parabola with the equation of directrix as

a. $x+a=0$ *b.* $2x+a=0$
c. $x=0$ *d.* $x=\frac{a}{2}$

125. The tangents drawn at the extremities of a focal chord of the parabola $y^2=16x$

a. intersect on $x=0$
b. intersect on the line $x+4=0$
c. intersect at an angle of 60°
d. intersect at an angle of 45°

126. On the set Z, of all integers $*$ is defined by $a*b=a+b-5$. If $2*(x*3)=5$, then $x=$

a. 0 *b.* 3
c. 5 *d.* 10

127. Which of the following is false?

a. Addition is commutative in N.
b. Multiplication is associative in N.
c. If $a*b=a^b$ for all $a, b\in N$, then $*$ is commutative in N.
d. Addition is associative in N.

128. If $\mathbf{a}\hat{i}=\mathbf{a}(\hat{i}+\hat{j})=\mathbf{a}(\hat{i}+\hat{j}+\hat{k})=1$, then $\mathbf{a}=$

a. $\hat{i}+\hat{j}$ *b.* $\hat{i}-\hat{k}$
c. $\hat{i}$ *d.* $\hat{i}+\hat{j}-\hat{k}$

129. If a and b are unit vectors and $|\mathbf{a}+\mathbf{b}|=1$, then $|\mathbf{a}-\mathbf{b}|$ is equal to

a. $\sqrt{2}$ *b.* 1
c. $\sqrt{5}$ *d.* $\sqrt{3}$

130. The projection of $\mathbf{a}=3\hat{i}-\hat{j}+5\hat{k}$ on $\mathbf{b}=2\hat{i}+3\hat{j}+\hat{k}$ is

a. $\frac{8}{\sqrt{35}}$ *b.* $\frac{8}{\sqrt{39}}$
c. $\frac{8}{\sqrt{14}}$ *d.* $\sqrt{14}$

131. If $f:R\to R$ is defined by $f(x)=x^3$, then $f^{-1}(8)=$

a. $\{2\}$ *b.* $\{2, 2\omega, 2\omega^2\}$
c. $\{2,-2\}$ *d.* $\{2,2\}$

132. R is a relation on N given by $R=\{(x,y)\,|\,4x+3y=20\}$. Which of the following belongs of R?

a. $(-4,12)$ *b.* $(5,0)$
c. $(3,4)$ *d.* $(2,4)$

133. If $\log_{10}7=08451$, then the position of the first significant figure of 7^{-20} is

a. 16 *b.* 17
c. 20 *d.* 15

134. $\frac{1}{2\cdot5}+\frac{1}{5\cdot8}+\frac{1}{8\cdot11}+\ldots$ upto n terms is equal to

a. $\frac{n}{4n+6}$ *b.* $\frac{1}{4n+4}$
c. $\frac{n}{6n+4}$ *d.* $\frac{n}{3n+7}$

135. The ten's digit in $1!+4!+7!+10!+12!+13!+15!+16!+17!$ is divisible by

a. 4 *b.* 3! *c.* 5 *d.* 7

136. The equation $\frac{x^2}{2-\lambda}-\frac{y^2}{\lambda-5}-1=0$ represents an ellipse if

a. $\lambda>5$ *b.* $\lambda<2$
c. $2<\lambda<5$ *d.* $2>\lambda>5$

137. The equation to the normal to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ at $(-4,0)$ is

a. $2x-3y=1$ *b.* $x=0$
c. $x=1$ *d.* $y=0$

138. The converse of the contrapositive of the conditional $p \to \sim q$ is

a. $p \to q$ *b.* $\sim p \to \sim q$

c. $\sim q \to p$ *d.* $\sim p \to q$

139. The perimeter of a certain sector of a circle is equal to the length of the arc of the semicircle. Then, the angle at the centre of the sector in radians is

a. $\pi - 2$ *b.* $\pi + 2$

c. $\frac{\pi}{3}$ *d.* $\frac{2\pi}{3}$

140. The value of $\tan 67\frac{1^\circ}{2} + \cot 67\frac{1^\circ}{2}$ is

a. $\sqrt{2}$ *b.* $3\sqrt{2}$

c. $2\sqrt{2}$ *d.* $2 - \sqrt{2}$

141. If e_1 and e_2 are the eccentricities of a hyperbola $3x^2 - 3y^2 = 25$ and its conjugate, then

a. $e_1^2 + e_2^2 = 2$ *b.* $e_1^2 + e_2^2 = 4$

c. $e_1 + e_2 = 4$ *d.* $e_1 + e_2 = \sqrt{2}$

142. If p and q are prime numbers satisfying the condition $p^2 - 2q^2 = 1$, then the value of $p^2 + 2q^2$ is

a. 5 *b.* 15

c. 16 *d.* 17

143. If $A(\text{adj } A) = 5I$, where I is the identity matrix of order 3, then $|\text{adj } A|$ is equal to

a. 125 *b.* 25

c. 5 *d.* 10

144. The number of solutions for the equation $\sin 2x + \cos 4x = 2$ is

a. 0 *b.* 1

c. 2 *d.* Infinite

145. $\int e^x \cdot x^5 \, dx$ is

a. $e^x[x^5 + 5x^4 + 20x^3 + 60x^2 + 120x + 120] + C$

b. $e^x[x^5 - 5x^4 - 20x^3 - 60x^2 - 120x - 120] + C$

c. $e^x[x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120] + C$

d. $e^x[x^5 + 5x^4 + 20x^3 - 60x^2 - 120x + 120] + C$

146. If $f(x)$ is an even function and $f'(x)$ exists, then $f'(e) + f'(-e)$ is

a. > 0 *b.* 0

c. ≥ 0 *d.* < 0

147. If α is a complex number satisfying the equation $\alpha^2 + \alpha + 1 = 0$, then α^{31} is equal to

a. α *b.* α^2

c. 1 *d.* i

148. The derivative of $\sin(x^3)$ w.r.t. $\cos(x^3)$ is

a. $-\tan(x^3)$ *b.* $\tan(x^3)$

c. $-\cot(x^3)$ *d.* $\cot(x^3)$

149. A unit vector perpendicular to both the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is

a. $\frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ *b.* $\frac{\hat{i} + \hat{j} - \hat{k}}{3}$

c. $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ *d.* $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

150. If $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $B = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ then

a. $A = -B$ *b.* $A = B$

c. $B = 0$ *d.* $B = A^2$

151. The locus of a point which moves such that the sum of its distances from two fixed points is a constant is

a. a circle *b.* a parabola

c. an ellipse *d.* a hyperbola

152. The centroid of the ΔABC, where $A \equiv (2, 3)$, $B \equiv (8, 10)$ and $C \equiv (5, 5)$ is

a. (5, 6) *b.* (6, 5) *c.* (6, 6) *d.* (15, 18)

153. If $3x^2 + xy - y^2 - 3x + 6y + k = 0$ represents a pair of lines, then $k =$

a. 0 *b.* 9 *c.* 1 *d.* -9

154. The equation of the smallest circle passing through the points (2, 2) and (3, 3) is

a. $x^2 + y^2 + 5x + 5y + 12 = 0$

b. $x^2 + y^2 - 5x - 5y + 12 = 0$

c. $x^2 + y^2 + 5x - 5y + 12 = 0$

d. $x^2 + y^2 - 5x + 5y - 12 = 0$

155. The characteristic roots of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ are

a. 1, 3, 6 *b.* 1, 2, 4

c. 4, 5, 6 *d.* 2, 4, 6

156. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^{-1} =$

a. $\frac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$ *b.* $\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

c. $\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$ *d.* $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

157. The set $\{-1, 0, 1\}$ is not a multiplicative group because of the failure of

a. Closure law *b.* Associative law
c. Identity law *d.* Inverse law

158. The angle of elevation of the top of a TV tower from three points A, B and C in a straight line through the foot of the tower are α, 2α and 3α respectively. If $AB = a$, then height of the tower is

a. $a\tan\alpha$ *b.* $a\sin\alpha$ *c.* $a\sin 2\alpha$ *d.* $a\sin 3\alpha$

159. The angles A, B and C of a ΔABC are in A.P. If $b : c = \sqrt{3} : \sqrt{2}$, then the angle A is

a. 30° *b.* 15° *c.* 75° *d.* 45°

160. $\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right)$ is equal to

a. $\frac{2\sqrt{126}}{65}$ *b.* $\frac{4\sqrt{65}}{65}$ *c.* $\frac{8\sqrt{63}}{65}$ *d.* $\frac{\sqrt{63}}{65}$

161. The general solution of $|\sin x| = \cos x$ is (when $n \in Z$) given by

a. $n\pi + \frac{\pi}{4}$ *b.* $2n\pi \pm \frac{\pi}{4}$
c. $n\pi \pm \frac{\pi}{4}$ *d.* $n\pi - \frac{\pi}{4}$

162. The real root of the equation $x^3 - 6x + 9 = 0$ is

a. -6 *b.* -9 *c.* 6 *d.* -3

163. The digit in the unit's place of 5^{834} is

a. 0 *b.* 1 *c.* 3 *d.* 5

164. The remainder when $3^{100} \times 2^{50}$ is divided by 5 is

a. 1 *b.* 2 *c.* 3 *d.* 4

165. $\int \frac{\sin x \cos x}{\sqrt{1 - \sin^4 x}}\,dx =$

a. $\frac{1}{2}\sin^{-1}(\sin^2 x) + C$ *b.* $\frac{1}{2}\cos^{-1}(\sin^2 x) + C$
c. $\tan^{-1}(\sin^2 x) + C$ *d.* $\tan^{-1}(2\sin x) + C$

166. The value of $\int_{-2}^{2}(ax^3 + bx + c)\,dx$ depends on the

a. value of b *b.* value of c
c. value of a *d.* value of a and b

167. The area of the region bounded by $y = 2x - x^2$ and the X-axis is

a. $\frac{8}{3}$ sq units *b.* $\frac{4}{3}$ sq units
c. $\frac{7}{3}$ sq units *d.* $\frac{2}{3}$ sq units

168. The differential equation $y\frac{dy}{dx} + x = c$ represents

a. a family of hyperbolas.
b. a family of circles whose centres are on the Y-axis.
c. a family of parabolas.
d. a family of circles whose centres are on the X-axis.

169. If $f(x^5) = 5x^3$, then $f'(x) =$

a. $\frac{3}{\sqrt[5]{x^2}}$ *b.* $\frac{3}{\sqrt[5]{x}}$
c. $\frac{3}{x}$ *d.* $\sqrt[5]{x}$

170. $f(x) = 2a - x$ in $-a < x < a$
$= 3x - 2a$ in $a \geq x$
Then which of the following is true?

a. $f(x)$ is discontinuous at $x = a$
b. $f(x)$ is not differentiable at x
c. $f(x)$ is differentiable at all $x \geq a$
d. $f(x)$ is continuous at all $x < a$

171. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is (in square units)

a. 4 *b.* 8π
c. 8 *d.* 5

172. If Z is a complex number such that $Z = -\overline{Z}$, then

a. Z is purely real.
b. Z is purely imaginary.
c. Z is any complex number.
d. Real part of Z is the same as its imaginary part.

173. The value of $\sum_{K=1}^{6}\left[\sin\frac{2K\pi}{7} - i\cos\frac{2K\pi}{7}\right]$ is

a. i *b.* 0
c. $-i$ *d.* -1

174. $\lim_{x\to\infty} x\sin\left(\frac{2}{x}\right)$ is equal to

a. ∞ *b.* 0
c. 2 *d.* $\frac{1}{2}$

175. A stone is thrown vertically upwards and the height x ft. reached by the stone in t sec is given by $x = 80t - 16t^2$. The stone reaches the maximum height in

a. 2 sec *b.* 2.5 sec
c. 3 sec *d.* 1.5 sec

176. The maximum value of $\frac{\log x}{x}$ in $(2, \infty)$ is

a. 1 *b.* $\frac{2}{e}$ *c.* e *d.* $\frac{1}{e}$

177. If $f(x) = be^{ax} + ae^{bx}$, then $f''(0) =$

a. 0 *b.* $2ab$

c. $ab(a + b)$ *d.* ab

178. If $\sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{x}{y}$, then the value of $\tan A =$

a. $\frac{x^2+y^2}{x^2-y^2}$ *b.* $\frac{2xy}{x^2+y^2}$

c. $\frac{2xy}{x^2-y^2}$ *d.* $\frac{2xy}{y^2-x^2}$

179. $\int \frac{\sec x}{\sec x + \tan x}\, dx =$

a. $\tan x - \sec x + C$ *b.* $\log(1 + \sin x) + C$

c. $\sec x + \tan x + C$ *d.* $\log\sin x + \log\cos x + C$

180. If $\int f(x)\, dx = g(x)$, then $\int f(x)\, g(x)\, dx =$

a. $\frac{1}{2} f^2(x)$ *b.* $\frac{1}{2} g^2(x)$

c. $\frac{1}{2}[g'(x)]^2$ *d.* $f'(x)\, g(x)$

ANSWERS

Physics

1. (c)	2. (d)	3. (b)	4. (a)	5. (d)	6. (d)	7. (b)	8. (d)	9. (c)	10. (a)
11. (b)	12. (a)	13. (d)	14. (a)	15. (a)	16. (b)	17. (d)	18. (b)	19. (d)	20. (a)
21. (a)	22. (d)	23. (b)	24. (b)	25. (c)	26. (b)	27. (c)	28. (d)	29. (d)	30. (b)
31. (c)	32. (b)	33. (a)	34. (a)	35. (a)	36. (d)	37. (d)	38. (a)	39. (c)	40. (c)
41. (b)	42. (b)	43. (c)	44. (d)	45. (b)	46. (c)	47. (a)	48. (a)	49. (a)	50. (c)
51. (c)	52. (c)	53. (b)	54. (c)	55. (b)	56. (c)	57. (a)	58. (c)	59. (a)	60. (b)

Chemistry

61. (a)	62. (c)	63. (c)	64. (b)	65. (c)	66. (c)	67. (d)	68. (c)	69. (b)	70. (c)
71. (a)	72. (b)	73. (b)	74. (d)	75. (d)	76. (d)	77. (b)	78. (c)	79. (c)	80. (a)
81. (b)	82. (d)	83. (a)	84. (a)	85. (b)	86. (d)	87. (a)	88. (c)	89. (a)	90. (a)
91. (d)	92. (b)	93. (c)	94. (*)	95. (a)	96. (c)	97. (a)	98. (a)	99. (d)	100. (a)
101. (a)	102. (c)	103. (d)	104. (c)	105. (d)	106. (b)	107. (a)	108. (d)	109. (c)	110. (b)
111. (b)	112. (d)	113. (c)	114. (b)	115. (c)	116. (a)	117. (c)	118. (d)	119. (c)	120. (a)

Mathematics

121. (d)	122. (a)	123. (c)	124. (c)	125. (b)	126. (d)	127. (c)	128. (c)	129. (d)	130. (c)
131. (a)	132. (d)	133. (b)	134. (c)	135. (b)	136. (b)	137. (d)	138. (d)	139. (a)	140. (c)
141. (b)	142. (d)	143. (b)	144. (a)	145. (c)	146. (b)	147. (a)	148. (c)	149. (d)	150. (b)
151. (c)	152. (a)	153. (d)	154. (b)	155. (a)	156. (a)	157. (d)	158. (c)	159. (c)	160. (a)
161. (c)	162. (d)	163. (d)	164. (d)	165. (a)	166. (b)	167. (b)	168. (d)	169. (a)	170. (d)
171. (c)	172. (b)	173. (a)	174. (c)	175. (b)	176. (d)	177. (c)	178. (c)	179. (a)	180. (b)

Note (*) None of the option is correct.

HINTS & SOLUTIONS

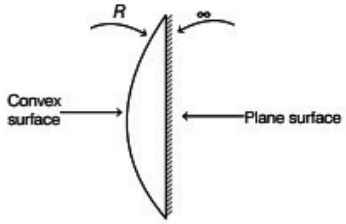
Physics

1. *(c)* Resultant focal length (f) of the given combination is 0.2 m.

Refractive index of the material, $\mu = 1.5$

From the lens Maker formula, we get

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



where, R_1 is the radius of curvature of convex surface and R_2 is the radius of curvature of plane surface ($R_2 = \infty$).

Hence,

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{0.2} = (0.5)\left(\frac{1}{R_1}\right)$$

$$\Rightarrow \quad R_1 = 0.5 \times 0.2 = 0.1 \text{ m}$$

2. *(d)* Dimensional formula of Planck's constant $= [M^1L^2T^{-1}]$

Similarly,

Boltzmann constant is given

$$K = \frac{\text{Energy}}{\text{Temperature}}$$

$\therefore$ The dimension of K is

$$= \frac{[ML^2T^{-2}]}{[K]} = [M^1L^2T^{-2}K^{-1}]$$

Dimensional formula of force is $[M^1L^1T^{-2}]$.

Dimensional formula of linear momentum is $[M^1L^1T^{-1}]$.

Dimensional formula of angular momentum is $[M^1L^2T^{-1}]$.

Hence, angular momentum and Planck's constant have same dimensional formula.

3. *(b)* Given, velocity of the balloon = 29ms^{-1}

When the stone is dropped, then the velocity of the stone

$$u = 29 \text{ms}^{-1}$$

Time taken to reach the ground, $t = 10$ s

From the second equation of motion,

$$s = -ut + \frac{1}{2}gt^2$$

$$= -29 \times 10 + \frac{1}{2} \times 9.8 \times (10)^2$$

$$= -290 + 490 = 200 \text{ m}$$

4. *(a)* The potential energy due to surface tension is given by the formula $U = S \times A$

where, S is the surface tension and A is the area of film. Since after puncturing, the film will try to minimise its potential energy and hence the surface area. Thus, the thread will become concave towards A.

5. *(d)* The velocity of sound in a gas at a fixed temperature is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where, $\gamma = \left(\frac{C_p}{C_V}\right)$ is specific heat ratio and M is the molecular mass of the gas.

Let, velocity of sound in hydrogen, $v_1 = \sqrt{\frac{\gamma RT}{M_1}}$

Velocity of sound in oxygen, $v_2 = \sqrt{\frac{\gamma RT}{M_2}}$

Let, M_1 and M_2 be the molecular mass of hydrogen and oxygen respectively n_1 and n_2 be the moles of hydrogen and oxygen, respectively.

Molecular mass of the mixture,

$$M_{\text{mix}} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

According to the question, $n_1 = n_2$ at given NTP.

$$M_2 = 16M \qquad \text{(given)}$$

$$M_{\text{mix}} = \frac{M_1 + 16M_1}{2} = \frac{17M_1}{2}$$

Velocity of sound in mixture, $v_{\text{mix}} = \sqrt{\frac{\gamma RT}{M_{\text{mix}}}}$

$$\Rightarrow \quad v_{\text{mix}} = \sqrt{\frac{2\gamma RT}{17M_1}}$$

$\therefore$ The ratio of velocity of sound in mixture to that of hydrogen

$$\frac{v_1}{v_{\text{mix}}} = \frac{\sqrt{\frac{\gamma RT}{M_1}}}{\sqrt{\frac{\gamma 2RT}{17M_1}}}$$

$$\Rightarrow \quad \frac{v_{\text{mix}}}{v_1} = \sqrt{\frac{2}{17}}$$

6. *(d)* Zero order principal maximum contains all incoming wavelets in diffraction pattern, Hence, zero order principal maximum will be white.

7. *(b)* H-polaroid is one of the most common type of polaroid used today. It is prepared by stretching a thin sheet of polyvinyl alcohol and then impregnating it with iodine. In this process, the molecules get oriented in the direction of the applied strain and the material becomes dichronic. Each polaroid sheet is mounted between thin glass plate to provide mechanical support.

8. *(d)* From Coulomb's law, we know

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$\Rightarrow \quad N = \frac{1}{\varepsilon_0}\frac{C^2}{m^2}$$

$$\Rightarrow \quad \varepsilon_0 = \frac{C^2}{m^2 N}$$

$$\Rightarrow \quad \varepsilon_0 = C^2 m^{-2} N^{-1}$$

9. *(c)* Given, capacitance of bigger drop = 1μF

Let R be the radius of bigger drop and r be the radius of smaller drops.

According to the question, $\frac{4}{3}\pi R^3 = 8\times\frac{4}{3}\pi r^3$

$$\Rightarrow \quad R^3 = 8r^3$$

$$\Rightarrow \quad R = 2r$$

The capacitance of bigger drop ,

$$C = 4\pi\varepsilon_0 R$$

$$= 2\times 4\pi\varepsilon_0 r \qquad (\because R = 2r)$$

Capacitance of smaller drop, $C' = 4\pi\varepsilon_0 r$

Thus, $\quad C' = \frac{C}{2}$

Hence, $\quad C' = \frac{1}{2}\mu F$

10. *(a)* The magnitude of the resultant force can be given by parallelogram's law of vector addition i.e.,

Resultant force $= \sqrt{p^2 + p^2 + 2p.p\cos\theta}$

$$\Rightarrow \quad F_{net} = \sqrt{2p^2 + 2p^2\times\cos(120°)} \quad (\because \theta = 120°)$$

$$= \sqrt{2p^2 + 2p^2\left(-\frac{1}{2}\right)}$$

$$= \sqrt{p^2}$$

$$= p$$

11. *(b)* Given that, amplitude and frequency of two waves are same.

Hence, resultant amplitude due to superposition of two waves with phase difference ϕ is given by

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$\Rightarrow A^2 = A^2 + A^2 + 2A^2\cos\phi \qquad (\because A_1 = A_2 = A)$$

$$\Rightarrow A^2 = 2A^2 + 2A^2\cos\phi$$

$$\Rightarrow \quad \cos\phi = \frac{-1}{2}$$

$$\Rightarrow \quad \phi = \frac{2\pi}{3}$$

12. *(a)* Given, time interval between echoes = 1 s

Speed of sound in air, $v = 340 ms^{-1}$

Let the distance between man and cliff be x.

Time taken by sound to hit cliff and return $= \frac{2x}{v}$

$$1 = \frac{2x}{v}$$

$$\Rightarrow \quad x = \frac{340}{2}\text{ m} = 170\text{ m}$$

Thus, distance between cliffs

$$= 170\times 2 = 340\text{ m}$$

13. *(d)* Given, wavelength of light beam,

$$\lambda = 600\text{ nm}$$

$$= 600\times 10^{-9}\text{ m}$$

Distance between slits, $d = 1\text{ mm} = 10^{-3}\text{ m}$

Distance between slits and screen, $D = 2\text{ m}$

Distance between first dark fringes from the central bright fringe, $2\beta = ?$

As we know, fringe width, $\beta = \frac{\lambda D}{d}$

$$= \frac{600\times 10^{-9}\times 2}{10^{-3}}$$

$$= 1.2\times 10^{-3}\text{ m}$$

Distance between first dark fringes on either side of central bright fringe $(2\beta) = 2\times\beta = 2\times 1.2\times 10^{-3}\text{ m}$

$$\Rightarrow \quad 2\beta = 2.4\text{ mm}$$

14. *(a)* Given, specific rotation of sugar solution,

$$\alpha = 0.01\text{ units}$$

Length of polarimeter tube, $l = 0.25$ m

Optical rotation, $\theta = 0.4$ rad

Concentration of solution, $c = ?$

As we know, specific rotation

$$= \frac{\text{Optical rotation}}{\text{Length of tube}\times\text{ Concentration of sol.}}$$

$$\alpha = \frac{\theta}{l\times c}$$

$$c = \frac{\theta}{l\times\alpha}$$

$$= \frac{0.4}{0.01\times 0.25}$$

$$= 160 kg / m^3$$

Concentration of sugar solution $= 200 kg / m^{-3}$ (given)

Percentage impurity of sugar in the sample

$$= \frac{160}{200}\times 100\%$$

$$= 80\%$$

15. *(a)* Given, potential difference, $V = 45.5$ V

Velocity, $v = ?$

Energy acquired by the electron after getting accelerated through the potential difference V is eV, which is equal to its kinetic energy.

Hence, kinetic energy (KE) $= \frac{1}{2}m_e v^2 = eV$

[where, m_e = mass of electron]

$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m_e}}$$

Substituting the given values, we get

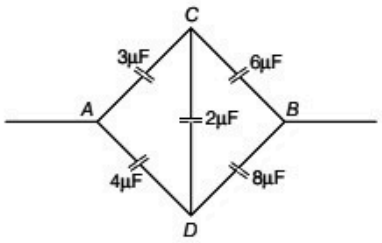
$$\Rightarrow \quad v = \sqrt{\frac{2\times 1.6\times 10^{-19}\times 45.4}{9.11\times 10^{-31}}}$$

$$= \sqrt{16\times 10^{12}} = 4\times 10^6 \text{ ms}^{-1}$$

16. *(b)* Earth is considered to be source of infinite charges having zero potential. When a body is connected to earth, it also acquires same potential as Earth i.e. zero. By this process excess charges flow through the body to the ground or the body acquires the opposite charges from the ground to make the body electrically neutral.

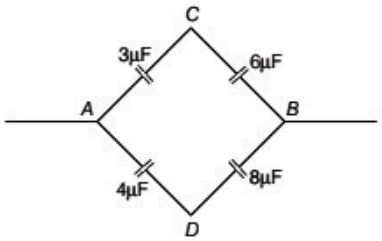
Thus, electrons flow from earth to the body to neutralise the positively charged body.

17. *(d)* The given combination is a balanced condition of Wheatstone bridge because potential at C and D points is same.



Hence, the 2μF capacitor can be neglected.

The equivalent circuit is given by the figure below



In the upper arm capacitors 3μF and 6μF are connected in series and similarly in lower arm capacitors 4 μF and 8 μF are connected in series . The equivalent capacitance is given by

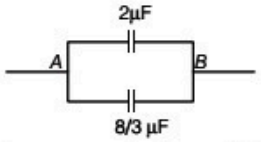
$$\frac{1}{C_1} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6}$$

$$\Rightarrow \quad C_1 = 2\mu\text{F}$$

and $$\frac{1}{C_2} = \frac{1}{4} + \frac{1}{8} = \frac{2+1}{8} = \frac{3}{8}$$

$$\Rightarrow \quad C_2 = \frac{8}{3}\mu\text{F}$$

The reduced circuit with C_1 and C_2 capacitance is given as



Now, C_1 and C_2 are connected in parallel combination. Hence, equivalent capacitance is given by

$$C_{eq} = C_1 + C_2$$

$$C_{eq} = 2 + \frac{8}{3} = \frac{14}{3}\mu\text{F}$$

18. *(b)* Radius of nth-orbit in hydrogen like atom is given by $$r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

i.e., $$r \propto \frac{n^2}{Z}$$

For hydrogen, $Z = 1$, $n = 1$ in ground state

$$\Rightarrow \quad \frac{n^2}{Z} = \frac{1^2}{1} = 1$$

For Beryllium (Be^{+++}), $Z = 4$ orbital is same

$$\therefore \quad \frac{n^2}{Z} = 1$$

$$\Rightarrow \quad n^2 = 1\times Z \Rightarrow n^2 = 4$$

$$\Rightarrow \quad n = 2$$

Thus, the second level of triply ionised Be^{+++} has same radius as the ground state of hydrogen.

19. *(d)* Packing fraction is equal to the ratio of mass defect to total number of nucleons.

i.e. PF $= \frac{\text{Mass defect}}{\text{No. of nucleons}} = \frac{M-A}{A}$

20. *(a)* Let N be the amount of substance left,

N_0 be the total amount of substance.

The amount of substance left after N half-life is given by $$\frac{N}{N_0} = \frac{1}{2^n}$$

Hence, $$\frac{30}{240} = \frac{1}{2^n}$$

(Since, decay rate proportional to amount of substance)

$$\Rightarrow \quad 2^n = 8 \Rightarrow n = 3$$

Number of half-life passed = 3

Time taken for the substance to decay from 240 count rate to 30 count rate = 1 h = 60 min

$$\text{Half-life} = \frac{\text{Total time}}{\text{Number of half - life}}$$

$$t_{1/2} = \frac{60}{3} = 20 \text{ min}$$

21. *(a)* Given, refractive index for blue light, $\mu_b = 1.67$

Refractive index for yellow light, $\mu_y = 1.65$

Refractive index for red light, $\mu_r = 1.63$

Dispersive power, $\omega = ?$

Dispersive power is the ratio of angular dispersion and mean deviation.

The formula of dispersive power is given as

$$\omega = \frac{\mu_b - \mu_r}{\mu_y - 1}$$

$$= \frac{1.67 - 1.63}{1.65 - 1}$$

$$= \frac{0.04}{0.65} = 0.0615$$

22. *(d)* Given, amount of heat absorbed, $Q = 6 \times 10^4$ J

Temperature of reservoir, $T_1 = 227°\text{ C} = 227 + 273\text{ K}$

$= 500\text{ K}$

Temperature of sink, $T_2 = 127°\text{ C}$

$= 127 + 273\text{ K}$

$= 400\text{ K}$

Efficiency of Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

$$= 1 - \frac{400}{500} = \frac{1}{5}$$

Work done by Carnot engine (W)

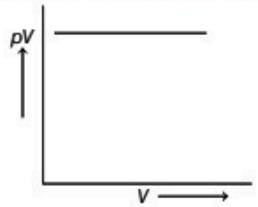
= Efficiency (η) × heat absorbed (Q)

$$\left[\because \eta = \frac{W}{Q}\right]$$

$$W = \frac{1}{5} \times 6 \times 10^4\text{ J} = 1.2 \times 10^4\text{ J}$$

23. *(b)* According to Boyle's law, at constant temperature, the pV relation will be constant. The pV *versus* p or V graph will be a straight line parallel to X-axis.

Hence, the pV *versus* V graph is given by



24. *(b)* Tiny water droplets suspended in air have greater density than air. The sunlight when changes medium, undergoes total internal reflection through these droplets thereby splitting in its component colours (since wavelength of each colour is different) and hence, results in rainbow formation.

Thus, dispersion and total internal reflection together results in rainbow formation.

25. *(c)* The combined focal length of plano-convex lens is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, $f_1 = \infty$ for the plane surface and $f_2 = f$

$$\therefore \quad \frac{1}{f} = \frac{1}{\infty} + \frac{1}{f}$$

$$\Rightarrow \quad f = f$$

Now, when concave lens of same focal length is joined to first lens, then combined focal length

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{1}{f} - \frac{1}{f} \qquad (\because F_1 = f, F_2 = -f)$$

$$\Rightarrow \quad \frac{1}{F} = 0 \Rightarrow F = \infty$$

Thus, the image can be focused on infinity or focus shifts to infinity.

26. *(b)* Ratio of diameters, $\frac{d_1}{d_2} = \frac{1}{2}$

Ratio of lengths, $\frac{l_1}{l_2} = \frac{2}{1}$

The rate of heat conduction is given by the formula,

$$H = \frac{dQ}{dt} = k\frac{A}{l}\Delta T$$

where, dQ is the amount of heat transferred in dt time, k is coefficient of heat conduction for a particular material. Since, ΔT is same in both cases, the ratio is

$$\frac{H_1}{H_2} = \frac{A_1 l_2}{A_2 l_1}$$

$$\Rightarrow \quad \frac{H_1}{H_2} = \frac{d_1^2 l_2}{d_2^2 l_1} = \frac{1}{4} \times \frac{1}{2} \Rightarrow R = 1:8$$

27. *(c)* The pressure inside our body is usually equal to the atomospheric pressure. Since, our mouth is open when blowing air, it will have the same pressure as the atmosphere, Hence, blowing air with open mouth is an example of isobaric process.

28. *(d)* Air is a completely elastic medium, i.e. it does not have modulus of rigidity, therefore sound wave in air are always longitudinal.

29. *(d)* White light consists inumerable wavelengths. If monochromatic light is replaced by white light in Young's double slit experiment, then each wavelength form their separate interference pattern. The resultant effect of all these patterns is obtained on the screen.

The waves of all colours reach at mid-point with same or zero phase difference. Therefore, central fringe is white. As fringe width i.e. wavelength increases in order of colours denoted by VIBGYOR, therefore on either side of it some coloured fringes are obtained in reverse order of VIBGYOR. After this the fringes of many colours overlap at each point of the screen and so the screen appears uniformly illuminated.

30. *(b)* Given, separation between two slits,

$$d = 0.9\text{ mm} = 0.9 \times 10^{-3}\text{ m}$$

Distance between slits and screen, $D = 1\text{ m}$

The distance between two consecutive dark or bright fringes is given as β (fringe width) and that between central fringe and first dark fringe on either side is $\frac{\beta}{2}$.

It is given that the spacing between second dark fringe and central fringe $= \beta + \frac{\beta}{2} = \frac{3\beta}{2}$

As we know, $\beta = \frac{\lambda D}{d}$

Hence, $\frac{3}{2} \times \frac{\lambda D}{d} = 1 \times 10^{-3}$ m

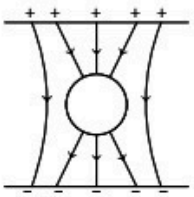
$\Rightarrow \frac{3}{2} \times \lambda \times \frac{1}{0.9 \times 10^{-3}} = 1 \times 10^{-3}$ m

$\Rightarrow \lambda = \frac{2}{3} \times 10^{-3} \times \frac{0.9 \times 10^{-3}}{1} = 0.6 \times 10^{-6}$ m

$\therefore$ Wavelength of monochromatic source of light,

$$\lambda = 600 \times 10^{-9} \text{ m} = 600 \text{ nm}$$

31. *(c)* When an uncharged sphere of metal is placed inside a charged parallel plate capacitor, charges of opposite polarity are induced on the surface of the sphere.



Negative charge would be induced on the upper surface of the sphere and similarly positive charge would be induced on lower surface of the sphere.

We know that, electric lines of force are always perpendicular to the surface and emerge from positive charge and enter into negative charge. Thus, the correct representation of the lines of force is given in the above figure.

32. *(b)* Given, resistance of the wire, $R = 6\Omega$

When it is cut in two equal parts, then resistance of the each part of wire is given by

$$r = \frac{R}{2} = 3\Omega \qquad (\because R \propto l)$$

Now, these new resistances are connected in parallel combination. The equivalent resistance is given by

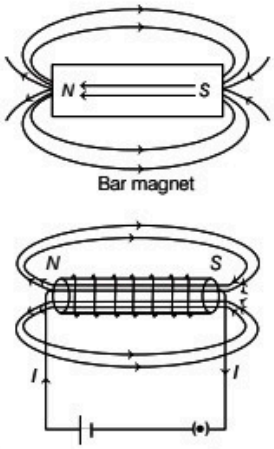
$$\frac{1}{R_{eq}} = \frac{1}{r} + \frac{1}{r}$$

$$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow \quad R_{eq} = 1.5\,\Omega$$

33. *(a)* The direction of magnetic field in a current carrying straight wire is given by right hand thumb rule. If we point our thumb in a direction of East to West, the direction of magnetic field is found to be towards North, which is given by the curled fingers of our hand.

34. *(a)* Magnetic field lines of a bar magnet is equivalent to the magnetic field lines due to a solenoid.



Hence, A bar magnet is equivalent to a solenoid carrying current.

35. *(a)* Excitation energy of hydrogen like ion in its first excitation state, $\Delta E = 40.8$eV

For the hydrogen like ion, energy in the ground state ($n = 1$),

$$E = \frac{-13.6Z^2}{n^2}\text{eV} = -13.6Z^2\text{eV}$$

Energy in the first excited state, $E_2 = \frac{-13.6Z^2}{n^2}$

$$= \frac{-13.6Z^2}{4}$$

Excitation energy, $\Delta E = E_2 - E_1$

$$= \left(\frac{-13.6Z^2}{4} + 13.6Z^2\right)\text{eV}$$

$$= (-3.4 + 13.6)Z^2\text{eV}$$

$$\Rightarrow \quad 40.8\text{eV} = 10.2Z^2\text{eV}$$

$$\Rightarrow \quad Z^2 = \frac{40.8}{10.2}$$

$$\Rightarrow \quad Z^2 = 4$$

$$\Rightarrow \quad Z = 2$$

Thus, energy of the electron in the ground state is given as $E_1 = -13.6Z^2\text{eV}$

$$\Rightarrow \quad E_1 = -13.6(2)^2\text{eV}$$

$$\Rightarrow \quad E_1 = -54.4 \text{ eV}$$

Hence, the energy needed to remove the electron from the ion in ground state (ionisation energy)

$$= -(-54.4)\text{eV} = 54.4\text{eV}$$

36. *(d)* Threshold wavelength of the metal, $\lambda_0 = 5200$Å

$$= 5200 \times 10^{-10} \text{ m}$$

Frequency of the radiation, $\nu_0 = \frac{c}{\lambda}$

$$= \frac{3\times10^8}{5200\times10^{-10}}$$
$$= 057\times10^{15}\ \text{Hz}$$
$$= 5.7\times10^{14}\ \text{Hz}$$

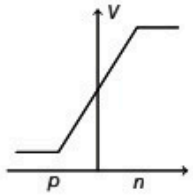
The frequency of infrared radiation is lower than the threshold frequency of the metal. Hence, the surface of the metal is illuminated with monochromatic radiation which is greater than 5.7×10^{14} Hz.

The 50 W UV-lamp is suitable for this purpose.

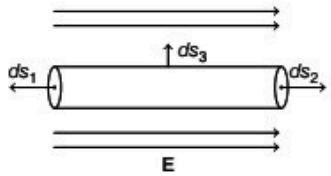
37. *(d)* The emitter is always forward biased to enable the majority charge carriers to cross the emitter-base junction and collector-base function is in reverse biased to collect the charge carriers, so that current flows through the transistor.

Hence, the emitter-base junction of a transistor is forward biased, while collector-base junction is reverse biased.

38. *(a)* Potential across the *p-n* junction varies symmetrically linear, having *p*-side negative and *n*-side positive. In a forward biased *p-n* junction diode, the repulsion of holes and electrons takes place which decreases width of potential barrier by striking the combination of holes and electrons. The width of depletion region in forward biased is less than the reversed biased. Hence, the graph of the potential barrier in the depletion region is given by



39. *(c)* The given diagram represents a cylinder placed in a uniform electric field.



Let ϕ_1 and ϕ_2 be the flux coming out of a circular cross-sectional surface. ϕ_3 is flux through the curved surface. Since, normal to a curved surface is always 90° to electric field E , so

$\phi_3 = Eds_3\cos\theta = Eds_3\cos 90° = 0$

ϕ_1 and ϕ_2 are equal and opposite to each other, hence $\phi_1 + \phi_2 = 0$.

So, total flux through the surface of cylinder is zero.

40. *(c)* Given, power of bulb *A*, $P_A = 60$ W

Power of bulb *B*, $P_B = 100$ W

Since, both bulbs are connected to the same power source in parallel combination. Thus,

$$P_A = \frac{V^2}{R_A} = 60\text{W and } P_B = \frac{V^2}{R_B} = 100\,\text{W}$$

$$R_A = \frac{V^2}{60}\Omega \text{ and } R_B = \frac{V^2}{100}\Omega$$

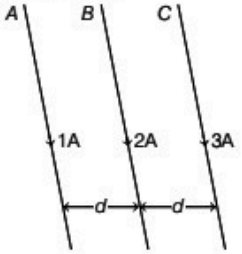
Current in bulb *A*, $I_A = \frac{\text{Potential}}{\text{Resistance}}$

$$\Rightarrow \quad I_A = \frac{V}{\frac{V^2}{60}} \Rightarrow I_A = \frac{60}{V}$$

Similarly, current in bulb *B*, $I_B = \frac{V}{\frac{V^2}{100}} = \frac{100}{V}$

It is clear that $I_B > I_A$.

41. *(b)* Magnetic field at any point due to a straight current carrying wire is given by



$$B = \frac{\mu_0 I}{2\pi d}$$

where, *d* is the distance of the point from the wire and I_1 is current flowing in the wire.

Force due to this magnetic field on current carrying wire is given by

$$F = BIL$$

where, *L* is the length of the wire.

$\therefore$ Force on wire *B* due to wire *A*, $F_A = BIL$

$$\Rightarrow \quad F_A = \frac{\mu_0\times1}{2\pi d}\times2\times L$$

Force on wire *B* due to wire *C*, $F_C = BIL$

$$\Rightarrow \quad F_C = \frac{\mu_0\times3}{2\pi d}\times2\times L$$

It is clear that $F_C > F_A$. Hence, net force is towards wire *C*.

42. *(b)* Ferromagnetic substances like iron, nickel and cobalt obey Curie-Weiss law above Curie temperature.

43. *(c)* EMF induced in circular coil of self-inductance *L* is given by $\varepsilon = -L\frac{dI}{dt}$

EMF is equal to the potential difference dimensionally.

Hence, $\varepsilon = V$

We know, $V = \frac{W}{q}$

Thus, dimensional formula of potential difference is given as

$$V = \frac{[M^1L^2T^{-2}]}{[AT]}$$
$$= [M^1L^2T^{-3}A^{-1}]$$

The self-inductance is given by, $L = V \times \frac{dt}{dI}$

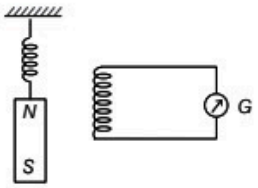
Dimensional formula of self-inductance is given by,

$$L = [M^1L^2T^{-3}A^{-1}] \times \frac{[T]}{[A]}$$

Dimensional formula of self-inductance,

$$L = [M^1L^2T^{-2}A^{-2}]$$

44. *(d)* The magnet is attached to the spring. This magnet will execute simple harmonic motion. When a coil is placed in such a way that magnet moves in and out of the coil, an emf is induced in the coil which opposes the movement of magnet in the coil. Due to this induced emf galvanometer will show deflections to the left and right periodically.



As the emf is induced in the coil eddy currents are also induced in the magnet, which lowers down the total magnetic strength of the magnet. Due to this reason, the magnetic flux linked with the coil decrease and hence amplitude of the deflections produced in the galvanometer also decreases.

45. *(b)* Given, resistance of the galvanometer, $G = 40\Omega$

Current in the galvanometer,

$$I_g = 10 \text{ m A} = 10 \times 10^{-3} \text{ A}$$

Maximum reading of voltmeter, $V = 50$ V

The galvanometer can be converted into voltmeter by connecting a shunt resistance (S) in the series.

Voltage in the voltmeter is given by

$$V = I_g(G + R)$$
$$\therefore \quad 50 = 10 \times 10^{-3}(40 + R)$$
$$\Rightarrow \quad 5000 = 40 + R$$
$$\Rightarrow \quad R = 4960\Omega$$

46. *(c)* R_1 is connected with 10Ω resistance. According to question, a balanced point is obtained at 50.

Thus, $\frac{R_1 + 10}{R_2} = \frac{50}{50}$

$$\Rightarrow \quad R_2 = R_1 + 10 \quad \ldots(i)$$

When 10Ω is removed the balanced point shifts to 40 cm.

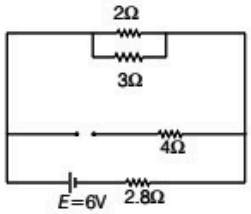
Thus, $\frac{R_1}{R_2} = \frac{40}{60}$

$$\Rightarrow \quad R_2 = \frac{3}{2}R_1 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{3}{2}R_1 = R_1 + 10$$
$$\Rightarrow \quad \frac{1}{2}R_1 = 10$$
$$\Rightarrow \quad R_1 = 20\Omega$$

47. *(a)* In the steady state analysis capacitor acts as an open circuit. Therefore, the equivalent circuit is given as



Thus, current in the 4Ω resistor is zero. 2Ω and 3Ω resistors are parallel to each other. Thus, the resistance in the upper branch is given by

$$R = \frac{2 \times 3}{2 + 3} = \frac{6}{5}\Omega$$

The upper branch resistor are connected to 2.8Ω resistor in series. Therefore, equivalent resistance in the circuit is given by

$$R_{eq} = 2.8 + \frac{6}{5}$$
$$= \frac{14.0 + 6}{5} = 4\Omega$$

Current, $I = \frac{\text{Voltage}}{\text{Resistance}} = \frac{6}{4} = 1.5$ A

Current in the 2Ω resistor is given by current division rule

i.e. $\quad I_{2\Omega} = I \times \frac{3\Omega}{2\Omega + 3\Omega}$

$$\Rightarrow \quad I_{2\Omega} = 1.5 \times \frac{3}{5} = 0.9\text{A}$$

48. *(a)* Given, area of rectangular coil, $A = 25 \text{ cm} \times 10 \text{ cm}$

$$\Rightarrow \quad A = 250\text{cm}^2 = 0.025\,\text{m}^2$$

No. of turns in the coil, $N = 300$

Magnetic field strength, $B = 4 \times 10^{-2}$T

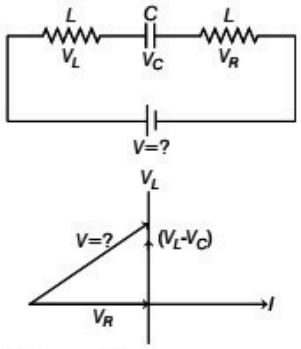
Peak voltage will be induced when the longer side of the coil moves perpendicular to the magnetic field.

$$\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$$

Induced emf is given by

$$\varepsilon = NBA\omega$$
$$= 300 \times 40 \times 10^{-2} \times 0.025 \times 100\pi = 3\pi\,\text{V}$$

49. *(a)* The *L-C-R* circuit in series combination and phasor diagram is given below



According to the question,

Potential difference across resistor, $V_R = 40\,V$

Potential difference across inductor, $V_L = 60\,V$

Potential difference across capacitor, $V_C = 30\,V$

Magnitude of potential difference supplied is given by

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{40^2 + (60-30)^2}$$

$$= \sqrt{(40)^2 + (30)^2} = \sqrt{2500}$$

$$= 50\,V$$

50. *(c)* Given, radius of the coil = 0.1 m

No. of turns in the coil = 10

Horizontal component of magnetic field,

$$B_H = 0.314 \times 10^{-4}\,T$$

The magnetic field at the centre of current carrying coil is given by the formula, $B = \frac{\mu_0 nI}{2R}$

According to question, magnetic field due to the coil obtains neutral point, in earth's magnetic field.

Hence, horizontal component of magnetic field,

$$B_H = \frac{\mu_0 nI}{2R}$$

Thus, $$I = \frac{2RB_H}{\mu_0 n}$$

$$\Rightarrow \quad I = \frac{2(0.1)(0.314 \times 10^{-4})}{10 \times 4 \times (3.142) \times 10^{-7}} = 0.5\,A$$

51. *(c)* The spectrum obtained from the chromosphere of the sun at the time of total is line emission spectrum. The spectrum obtained is of 5876Å wavelength from D_3 line in almost yellow colour.

52. *(c)* Heavy water is compound that contains heavy hydrogen or deuterium and oxygen. Heavy water is also called deuterium oxide D_2O.

53. *(b)* The nuclear reactor at Kaiga is a power reactor. The use of nuclear reactor for electricity and power generation classifies it as a power reactor.

54. *(c)* The work done by a force in moving a body is

$$W = \mathbf{F} \cdot \mathbf{s}$$

where, s is the displacement vector.

In circular path, the force is centripetal, thus always acting towards the centre along the radial direction.

However, the displacement is always along tangent to the circle at the instantaneous position of object. Hence, $\theta = 90°$

Thus, $$W = \mathbf{F} \cdot \mathbf{s} = Fs\cos 90° = 0$$

55. *(b)* Given, velocity of the bullet $v = 100\,ms^{-1}$

Let, thickness of one plank, $t = s$

According to third equation of motion, we get

$$v^2 - u^2 = 2as$$

$$(0) - (100)^2 = 2 \times a \times (2s)$$

(∵ two planks are penetrated)

$$\Rightarrow \quad -1000 = 4as \qquad \ldots(i)$$

When velocity of bullet is doubled, $v = 200\,ms^{-1}$

Applying third equation of motion again , we get

$$\Rightarrow \quad 0 - (200)^2 = 2 \times a \times (ns)$$

(where, n is number of planks)

$$-40000 = 2ans \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{-40000}{-10000} = \frac{2ans}{4as}$$

$$\Rightarrow \quad n = 8$$

Hence, 8 planks can be penetrated by the same bullet if speed is doubled.

56. *(c)* Let m_1 be the mass of 1st body , $m_1 = 1$ kg

m_2 be the mass of 2nd body, $m_2 = 2$ kg

The relation, between kinetic energy and momentum is given by

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

where, p is the momentum of the body. Since, momentum of both bodies are same.

Hence, $$\frac{k_1}{k_2} = \frac{p^2}{2m_1} \times \frac{2m_2}{p^2}$$

$$\frac{k_1}{k_2} = \frac{2}{1}$$

Thus, the ratio of their kinetic energies is 2 : 1.

57. *(a)* Pitch is dependent on the frequency of the source of sound whereas loudness depends upon the amplitude of sound wave.

Hence, the loudness and pitch of a sound note depends on intensity and frequency.

58. *(c)* Open windows and doors are considered to be perfect black body, hence absorption coefficient of an open window and door is 1.

59. *(a)* In Melde's experiment, the variation in frequency is created if the tension is increased in the string. Melde's showed that mechanical waves also undergo phenomenon of interference. To do this a resonance in tuning fork and frequency of standing waves was developed.

Hence, the frequency of the tuning fork and the frequency of the waves in the string are in the ratio 1 : 1.

60. *(b)* Given, speed of sound in air, $c = 300\,ms^{-1}$

Difference between apparent frequency during approach and apparent frequency during recession = 2%

Thus, frequency during approach – frequency during recession $= \frac{2}{100} \times f_0$

$$\Rightarrow \quad f_0\left(\frac{c+v}{c}\right) - f_0\left(\frac{c-v}{c}\right) = \frac{2}{100} f_0$$

where, c is speed of sound in air and v is the velocity of source.

$$\Rightarrow \quad \frac{2v}{300} = \frac{2}{100}$$

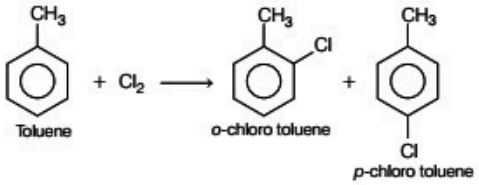
$$\Rightarrow \quad v = 3\,ms^{-1}$$

Chemistry

61. *(a)* Ozone in stratosphere is depleted by number of pollutants like CFC's (CF_2Cl_2), nitrogen oxide, CCl_4, Cl_2 etc.

62. *(c)* Linolenic acid is an unsaturated fatty acid. It is a carboxylic acid with 18- carbon chain having three double bond.

63. *(c)* When chlorine is passed through boiling toluene, mixture of *o* and *p*-chloro toluene is formed.



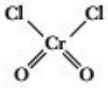
64. *(b)* The standard temperature used in thermo chemical calculation is 25° C or 298 K

$$K = T + 273 = 25 + 273 = 298\,K$$

65. *(c)* Density is an intensive property. It is independent of the quantity of substance. While, enthalpy, entropy and mass are extensive properties.

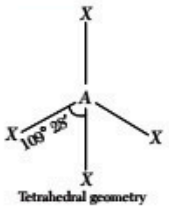
66. *(c)* Schiff's reagent contains rosaniline ($C_{19}H_{18}N_3Cl$). It is used to check the presence of aldehyde in the given compound by giving purple or magneta colour.

67. *(d)* Chromyl chloride is inorganic compound with formula CrO_2Cl_2. It is reddish compound that is volatile at room temperature. It's structure is



68. *(c)* Horn silver is a halide ore. The formula for horn silver is AgCl.

69. *(b)* Tetrahedral structure is formed by sp^3 hybridisation. Here, four sp^3-hybridised orbitals are arranged in tetrahedral structure.



Tetrahedral geometry

70. *(c)* NO^+ ligand is called nitrosonium ligand. It is positive type of ligand, which donate pair of electrons to central metal atom and forms coordinate bond with them.

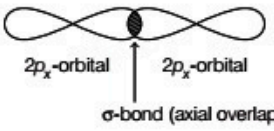
71. *(a)* A compound in which the complex ion carries positive charge is called cationic complex.

For hexaamino platinum chloride, chemical formula is $[Pt(NH_3)_6]Cl_4$.

$\therefore$ The complex ion $[Pt(NH_3)_6]^{4+}$ is positively charge.

72. *(b)* $2p_x$ atomic orbital undergoes linear combination with $2p_x$-orbital.

For linear combination, the combining orbitals should have same energy and same orientation of overlapping of p_x-orbital.



73. *(b)* $[A_0] = 0.1$

$$[A_t] = 0.01$$

$$t = 100\,s$$

1st order reaction,

$$k = \frac{2.303}{t} \log \frac{[A_0]}{[A_t]}$$

$$= \frac{2.303}{100} \log \frac{0.1}{0.01} = 0.02303\,s^{-1}$$

74. *(d)* When Δn_g i.e. difference in gaseous moles of product and reactant is zero, then pressure has no effect and it is equilibrium point for reaction.

$$N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$$

$$\Delta n_g = 2 - (1 + 1) = 0$$

$\therefore$ Pressure has no effect for this reaction.

75. *(d)* Conductivity of a solution is not affected by addition of ethanol as it is a covalent compound and it do not dissociate into ions to change the number of ions in the solution.

76. *(d)* Lowering in vapour pressure is a colligative property so, the number of particle (i.e. ion) increase, which results in decrease of vapour pressure. The lowering in vapour pressure is maximum for 0.1M $K_4[Fe(CN)_6]$, as it dissociate into free ions i.e. four K^+ and one $[Fe(CN)_6]^-$.

77. *(b)* The given reaction takes place as follows :

$$\underset{\text{Bromoethane}}{CH_3—CH_2Br} + Cl—\underset{\underset{\text{Isopropyl chloride (2-chloro propane)}}{CH_3}}{\overset{|}{CH}}—CH_3 \xrightarrow[\text{Na in ether}]{} CH_3CH_2—\underset{\underset{\text{2-methyl butane}}{CH_3}}{\overset{|}{CH}}—CH_3$$

78. *(c)* NH_3 (ammonia) gas is dried by using sodalime, because NH_3 itself is basic in nature and only basic compound can be used to dry NH_3, while conc. H_2SO_4, anhy. $CaCl_2$ and P_2O_5 can not react with ammonia.

79. *(c)* Copper (II) hydroxide $[Cu(OH)_2]$ is amphoteric in nature and is soluble in excess of ammonium hydroxide.

$$\underset{\text{Copper (II) hydroxide}}{Cu(OH)_2} + 4\underset{\text{Ammonium hydroxide}}{NH_4OH} \longrightarrow \underset{\text{Copper tetra ammine dihydroxide (Soluble complex)}}{[Cu(NH_3)_4](OH)_2} + 4H_2O$$

80. *(a)* Potassium dichromate $(K_2Cr_2O_7)$ can be converted into potassium chromate (K_2CrO_4) by adding potassium hydroxide (KOH).

$$\underset{\text{Potassium dichromate}}{K_2Cr_2O_7(aq)} + \underset{\text{Potassium hydroxide}}{2KOH} \longrightarrow \underset{\text{Potassium chromate}}{2K_2CrO_4} + H_2O$$

81. *(b)* Equivalent moles of NaOH = Equivalent moles of acid

$$N_1V_1 = \frac{\text{Weight}}{\text{Equivalent mass}}$$

$$40 \times 10^{-3} \times 0.125 = \frac{0.5}{\text{Equivalent mass}}$$

$\therefore$ Equivalent mass of acid = 100

82. *(d)* $M_1 = 5$ L and pH = 12

$$pOH = 14 - pH = 14 - 12 = 2$$

$$[OH^-] = 10^{-pOH} = 10^{-2}$$

$\therefore$ Moles of NaOH = $M_1V_1 = 5 \times 10^{-2}$ moles

$\therefore$ Mass of NaOH = moles $\times$ molar mass

$$= 5 \times 10^{-2} \times 40 = 2\,g$$

83. *(a)* Equivalent moles of oxalic acid = Equivalent moles of $KMnO_4$

$$= N_1V_1 = 25 \times 0.2 \times 10^{-3} = 5 \times 10^{-3} = 0.005$$

$$\frac{\text{Weight of oxalic acid}}{\text{Eq. weight of oxalic acid}} = 0.005$$

$$\text{Weight of oxalic acid} = 0.05 \times 63 = 0.315\ g$$

$\therefore$ 0.315 g in 50 cc of oxalic acid.

In 500cc solution, mass of oxalic acid is

$$= 0.315 \times \frac{500}{50} = 3.15\ g$$

84. *(a)* Pure water is neutral as it has equal concentration of H^+ and OH^- ions. It has pH = 7.

85. *(b)* In the titration of Mohr's salt with $KMnO_4$, the indicator used is $KMnO_4$ itself. Because $KMnO_4$ is very intense purple in colour. When, the solution changes to pink, end point is noted.

$$\underset{\text{Dark purple}}{MnO_4^-} \xrightarrow[\text{Mohr's salt}]{\text{Reduction with}} \underset{\text{Pink}}{Mn^{2+}}$$

86. *(d)* The relationship between half-life of a reaction and order of reaction is

$$t_{1/2} \propto \frac{1}{a^{(n-1)}}$$

87. *(a)* $\frac{p^\circ - p_S}{p^\circ} = \frac{w_2M_1}{M_2w_1}$

Where, w_2 and M_2 are weight and molecular mass of solute.

M_1 and w_1 are molecular mass and weight of solvent.

$$\frac{\Delta p}{p^\circ} = \frac{6}{60} \times \frac{18}{90} = \frac{1}{50} = 0.02$$

88. *(c)* Molar mass of sucrose $(C_{12}H_{22}O_{11}) = 342$

$$\text{Molality} = \frac{\frac{\text{Weight of sucrose}}{\text{Molar mass of sucrose}}}{\text{Mass of solvent (g)}} \times 1000$$

$$= \frac{6.84 \times 1000}{342 \times 200} = 0.1\ M$$

89. *(a)* NaCl on dissociation gives Na^+ and Cl^- ions as follows:

$$\underset{\text{Sodium chloride}}{NaCl(aq)} \longrightarrow \underset{\text{Common ions}}{Na^+} + Cl^-\ (\text{ions})$$

While, soap in the solution gives $RCOO^-$ and Na^+ ions as follows:

$$\underset{\text{Soap}}{R—\overset{-}{C}OO\overset{+}{N}a(s)} \longrightarrow RCOO^- + Na^+$$

$\therefore$ Solubility of soap decrease due to common ion (Na^+) present in the solution due to Le-Chatelier's principle, reaction shifts towards the reactant side.

90. *(a)* Concentrations of ions in 0.1 M aluminium sulphate $[Al_2(SO_4)_3]$ is

$$= 0.1[2 \times (Al^{3+})\text{ions} + 3 \times (SO_4^{2-})\text{ions}]$$

$$= 0.1\,[2 \times 1 + 3 \times 1] = 0.1 \times 5 = 0.5\ M$$

Concentrations of ions in 0.1 M potassium nitrate (KCl) is = 0.1 (1 mole of K^+ + 1 mole of Cl^-)

$$= 0.1 \times 2 = 0.2\ M$$

Concentration of ions in 0.1 M magnesium chloride $(MgCl_2)$ is = 0.1 (1 mole of Mg^{2+} + 2 moles of Cl^-)

$$= 0.1 \times 3 = 0.3\ M$$

Concentration of ions in 0.1 M barium chloride $(BaCl_2)$ is = 0.1 (1 mole of Ba^{2+} + 2 moles of Cl^-)

$$= 0.1 \times 3 = 0.3\ M$$

$\therefore$ Order of concentration is

0.1 M $Al_2(SO_4)_3$ > 0.1 M KCl > 0.1 $MgCl_2$ = 0.1 M $BaCl_2$

$\because$ Osmotic pressure is colligative property and, it depends on concentration of ions.

∴ Order of osmotic pressure is

$0.1\ M\ (Al_2SO_4)_3 > 0.1\ M\ KCl > 0.1\ M\ MgCl_2 > 0.1\ M\ BaCl_2$

91. *(d)* As reaction is completed 50% so, $t_{1/2} = 30$ min.

For first order reaction

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{30} = 0.0231 \text{min}^{-1}$$

92. *(b)* The ebullioscopic constant (K_b) is the elevation in boiling point produced by 1 molal solution

i.e. $\Delta T_b = K_b\, m$

When, $m = 1$

$K_b = \Delta T_b$

93. *(c)* Molecular mass of glucose = 180 g / mol

Mass of water = 50 g = 0.05 kg

$$\text{Molality}\,(m) = \frac{\text{Mass}}{\text{Molar mass} \times \text{mass of solvent (in kg)}}$$

$$0.3 = \frac{\text{Mass}}{160 \times 0.05}$$

Mass = 2.7 g

94. *(*)* The reaction of Mohr's salt and $K_2Cr_2O_7$ is as follow:

$6Fe^{2+} + Cr_2O_7^{2-} + 14[H^+] \longrightarrow 6Fe^{3+} + 2Cr^{3+} + 7H_2O$

n-factor of $Cr_2O_7^{2-}$ is 6 and *n*-factor of Fe^{2+} is 1.

As *n*-factor for Fe is 1 so,

Molarity = Normality Concentration of Mohr's salt = 0.08 M

Moles in 25 mL of 0.08 M Mohr's salt

$$= 0.08 \times \frac{25}{1000} = 0.002$$

∴ Moles in 500 cc of Mohr's salt

$$= 0.002 \times \frac{500}{25} = 0.04$$

Hence, mass of Mohr's salt in 500 cc

$= 0.04 \times 392 = 15.68$ g

95. *(a)* For a reaction to be spontaneous change in Gibbs free energy should be negative.

i.e. $\Delta G = \Delta H - T\Delta S < 0$

$= (-\text{ve}) - (+\text{ve}) = -\text{ve} - \text{ve} = -2\text{ve}$

$\Delta H < 0$ and $\Delta S > 0$

So, $\Delta H = -$ ve and $\Delta S = +$ve

96. *(c)* Sodium chloride is in simple closed cubic packing. Its coordination number is 6.

97. *(a)* Conjugate acid of NH_2^- is NH_3.

$$\underset{\text{Base}}{NH_2^-} + H^+ \longrightarrow \underset{\text{Conjugate acid}}{NH_3}$$

98. *(a)* Molar conductivity increases with decrease in concentration as the total volume, (V) of a solution containing one mole of electrolyte also increases.

∴ 0.005 M NaCl have highest molar conductivity.

99. *(d)* In the detection of III group, basic radicals NH_4OH is added after NH_4Cl to decrease the ionisation of NH_4OH due to common ion effect.

100. *(a)* Just before attaining chemical equilibrium rate of forward reaction decrease and rate of backward reaction increases.

As the concentration of product is slightly more than the equilibrium concentration.

101. *(a)* Atomic number of Fe (26) = $[Ar]3d^6 4s^2$

$Fe^{2+} = [Ar]3d^6 4s^0$

⇅	1	1	1	1

$3d^6$

Number of unpaired electrons = 4

For $Co^{2+} = [Ar]\ 3d^7$

Number of unpaired electrons = 3

It shows highest magnetic moment. As magnetic moment depends on number of unpaired electrons.

For $Cr^{3+} = [Ar]3d^3$

∴ Number of unpaired electrons = 3

For $Ni^+ = [Ar]3d^8$

∴ Number of unpaired electrons = 2

102. *(c)* In 3*d* series as we move from Sc to Zn paramagnetism first increases as number of unpaired electron increases . After the elements get half-filled the number of unpaired electron decreases as pairing take place.

∴ The number of unpaired electron, decreases thus, paramagnetism also decreases.

103. *(d)* Fe (26) = $[Ar]3d^6 4s^2$

$Fe^{3+}(23) = [Ar]3d^5 4s^0$

$Fe^{3+} =$

1	1	1	1	1

$3d^5$

∴ Number of unpaired electrons = 5

104. *(c)* In $K_4[Fe(CN)_6]$,

Oxidation number of Fe $\Rightarrow 4 + x + 6 \times (-1) = 0$

$x = +2$

It's IUPAC name is potassium hexacyanoferrate (II).

105. *(d)* Adsorption of an inert gases on activated charcoal increases with decrease of temperature as it is an exothermic process. The adsorption of gas on solid surface is similar to condensation process, which is exothermic in nature.

106. *(b)* Brine is a solution of sodium chloride and water. On electrolysis, it gives chlorine (Cl_2), hydrogen (H_2) and sodium hydroxide (NaOH).

At cathode

$$2H^+(aq) + 2e^- \longrightarrow \underset{\text{Hydrogen}}{H_2}$$

$$2H_2O + 2e^- \longrightarrow H_2 + 2OH^-(aq)$$

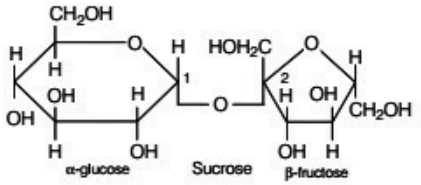
At anode

$$2Cl^- \longrightarrow \underset{\text{Chlorine}}{Cl_2} + 2e^-$$

The hydroxide ion, with the unchanged sodium ion, forms NaOH.

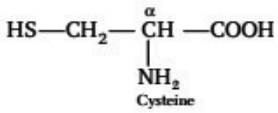
$$Na^+ + OH^- \longrightarrow \underset{\substack{\text{Sodium}\\\text{hydroxide}}}{NaOH}$$

107. *(a)* Sucrose is a non-reducing sugar due to 1, 2-glycosidic linkage between anomeric carbon atoms.

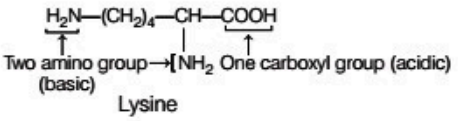


Therefore, there is no free aldehyde or ketone group (reducing of group) as they are involved in glycosidic linkage.

108. *(d)* Sulphur containing amino acid is cysteine. It's structure is



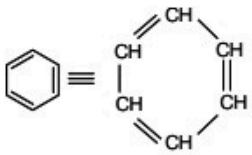
109. *(c)* Lysine is a basic amino acid as it contains more number of amino groups than carboxyl groups. It's structure is



110. *(b)* In Molisch's reagent substance used is α-napthol dissolved in ethanol. It gives positive test with carbohydrates, nucleic acids and glycoproteins by forming purple-coloured product.

111. *(b)* In benzene, each carbon have sp^2 hybridisation.

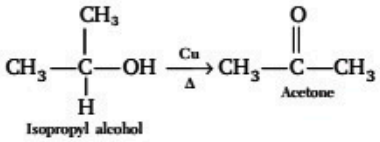
The structure of benzene is



Each carbon have three σ-bonds.

$\therefore$ Hybridisation = sp^2

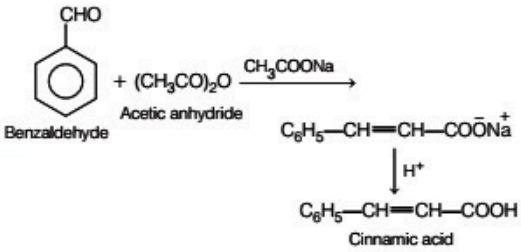
112. *(d)* Alcohols are oxidised by passing their vapours over heated copper. 2° alcohol (isopropyl alcohol) is oxidised to acetone i.e. it undergoes dehydrogenation as follows :



113. *(c)* The IUPAC name of  is

N, N-dimethyl methanamine.

114. *(b)* The reaction of benzaldehyde with acetic anhydride in presence of fused sodium acetate gives cinnamic acid. This reaction is called Perkin condensation.



115. *(c)* When a mixture of calcium benzoate and calcium formate is dry distilled, we get benzaldehyde.

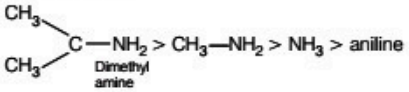
$$\underset{\text{Calcium benzoate}}{(Ph—COO)_2Ca} + \underset{\text{Calcium formate}}{(HCOO—)_2Ca} \xrightarrow[\text{Dry distillation}]{\Delta} \underset{\text{Benzaldehyde}}{2Ph—CHO} + 2CaCO_3$$

116. *(a)* Basic strength of amine increase with increases in strength of electron donating group. In dimethyl amine there are two methyl group with + *I*-effect and is most basic.

While, methyl amine has only one methyl group and ammonia has no methyl group.

In aniline due to resonance the lone pair of amines are delocalised and is least basic.

Basic strength order

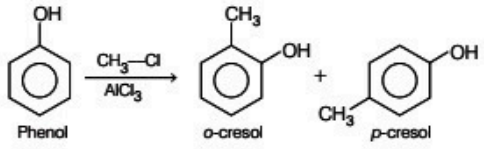


117. *(c)* Benzoic acid is bi-functional compound as it has two functional groups, carboxylic acid and aromatic ring.

Cinnamic acid is tri-functional compound as it has three functional groups, i.e., —COOH, C═C and aromatic ring.

Formic acid and acetic acid are mono-functional as they have one functional group i.e., (—COOH group).

118. *(d)* When phenol is treated with chloromethane in presence of $AlCl_3$ we get mixture of *o* and *p*-cresol. It is an example of Friedel-Craft's alkylation reaction.



119. *(c)* For the following reaction,

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

Δn = number of moles of product in gaseous state − number of moles of reactant in gaseous state

$= 2 - (3 + 1) = -2$

$\because \quad K_p = K_C(RT)^{\Delta n}$

$\therefore \quad K_p = K_C(RT)^{-2}$.

120. *(a)* According to Faraday's second law,

$$\frac{w_1}{E_1} = \frac{w_2}{E_2}$$

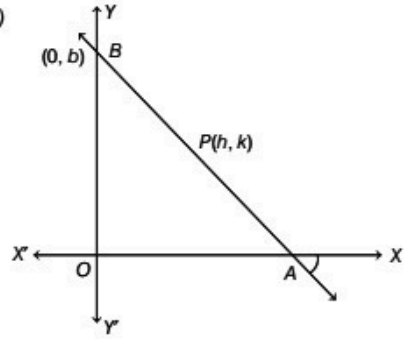
where, w and E are mass and equivalent weight respectively.

$$\frac{0.4}{\frac{63.5}{2}} = \frac{w_2}{108}$$

$\therefore \quad w_2 = 1.35$ g

Mathematics

121. *(d)*



We have, equation of line $\frac{x}{a} + \frac{y}{b} = 1$.

Let $P(h, k)$ be the required point.

Now, coordinates A and B are $(a, 0)$ and $(0, b)$ respectively. Since, P is mid-point of AB.

Therefore,

$$(h, k) = \left(\frac{a+0}{2}, \frac{0+b}{2}\right)$$

$\Rightarrow \quad h = \frac{a}{2}$ and $k = \frac{b}{2}$

$\Rightarrow \quad a = 2h$ and $b = 2k$

Now, it is given that

$$a + b = 4$$

$\Rightarrow \quad 2h + 2k = 4$

$\Rightarrow \quad h + k = 2$

So, locus of P is $x + y = 2$

122. *(a)* Let $S_1 : x^2 + y^2 - 8x = 0$

$$S_2 : x^2 + y^2 - 5x + 7y = 0$$

$$S_3 : x^2 + y^2 - 5x + 7y - 1 = 0$$

$$S_4 : x^2 + y^2 - 8x + 7y - 2 = 0$$

Now, at $(5, -7)$

$S_1 = 25 + 49 - 40 = 34 > 0$

$S_2 = 25 + 49 - 25 - 49 = 0$

$S_3 = 25 + 49 - 25 - 49 - 1 = -1 < 0$

$S_4 = 25 + 49 - 40 - 49 - 2 = -17 < 0$

$\therefore (5, -7)$ lies outside the circle S_1.

123. *(c)* Let $S_1 : x^2 + y^2 = 9$

and $S_2 : x^2 + y^2 + 2\alpha x + 2y + 1 = 0$

Now, centre and radius of S_1 are

Centre $= C_1(0, 0)$

Radius $= 3$

Centre and radius of S_2 are

Centre $= C_2(-\alpha, -1)$

Radius $= \sqrt{(-\alpha)^2 + (-1)^2 - 1} = \alpha$

Since, S_1 and S_2 touch each other internally, then

$$C_1C_2 = |r_2 - r_1|$$

$\Rightarrow \quad \sqrt{(-\alpha - 0)^2 + (-1 - 0)^2} = |\alpha - 3|$

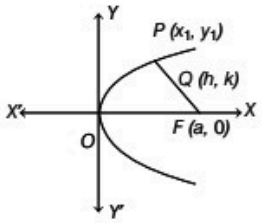
$\Rightarrow \quad \sqrt{\alpha^2 + 1} = |\alpha - 3|$

$\Rightarrow \quad \alpha^2 + 1 = (\alpha - 3)^2$

$\Rightarrow \quad \alpha^2 + 1 = \alpha^2 - 6\alpha + 9$

$\Rightarrow \quad 6\alpha = 8 \Rightarrow \alpha = \frac{4}{3}$

124. *(c)* Let the mid-point of any point $P(x_1, y_1)$ on parabola and focus $F(a, 0)$ be $Q(h, k)$.



Then, $(h, k) = \left(\frac{x_1 + a}{2}, \frac{y_1 + 0}{2}\right)$

$\Rightarrow \quad h = \frac{x_1 + a}{2}$ and $k = \frac{y_1}{2}$

$\Rightarrow \quad x_1 = 2h - a$ and $y_1 = 2k$

Since, $P(x_1, y_1)$ lies on parabola $y^2 = 4ax$, then

$$(2k)^2 = 4a(2h - a)$$

$$4k^2 = 4a(2h - a)$$

$$k^2 = 2a\left(h - \frac{a}{2}\right)$$

$\Rightarrow \quad y^2 = 2a\left(x - \frac{a}{2}\right)$

Equation of its directrix is

$$x - \frac{a}{2} = \frac{-a}{2}$$

$\Rightarrow \quad x = 0$

125. *(b)* The tangent at the extremities of a focal chord of a parabola intersect at right angles on the directrix.

Now, equation of parabola is $y^2 = 16x$.

So, its directrix is $x = -4$ or $x + 4$.

So, tangent intersect on the line $x + 4 = 0$.

126. *(d)* We have,

$a * 3 = a + b - 5$

Now, $x * 3 = x + 3 - 5 = x - 2$

Again, $2 * (x * 3) = 5$

$\Rightarrow \quad 2 * (x - 2) = 5$

$\Rightarrow \quad 2 + x - 2 - 5 = 5$

$\Rightarrow \quad x = 10$

127. *(c)* (a) $a + b = b + a, \forall\, a, b \in N$.

So, addition is commutative in N.

(b) $(a \times b) \times c = a \times (b \times c), \forall\, a, b, c \in N$.

So, multiplication is associative in N.

(c) $a * b = a^b$ and $b * a = b^a$

$\therefore \quad a * b \neq b * a; \forall\, a, b \in N$

So, $*$ is commutative in N is false.

(d) $(a + b) + c = a + (b + c), \forall\, a, b$ and $c \in N$.

So, addition is associative in N.

128. *(c)* Let $\mathbf{a} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

Now, $\mathbf{a} \cdot \hat{\mathbf{i}} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot \hat{\mathbf{i}} = x$

$\mathbf{a} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = x + y$

and $\mathbf{a} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = x + y + z$

Now, according to the question,

$$\mathbf{a} \cdot \hat{\mathbf{i}} = \mathbf{a}(\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \mathbf{a} \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$$

$\Rightarrow \quad x = x + y = x + y + z = 1$

$\Rightarrow \quad x = 1, y = 0, z = 0$

$\therefore \quad \mathbf{a} = \hat{\mathbf{i}}$

129. *(d)* We have

$$|\mathbf{a}| = |\mathbf{b}| = 1, |\mathbf{a} + \mathbf{b}| = 1$$

We know that,

$$|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2)$$

$\Rightarrow \quad (1)^2 + |\mathbf{a} - \mathbf{b}|^2 = 2((1)^2 + (1)^2)$

$\Rightarrow \quad 1 + |\mathbf{a} - \mathbf{b}|^2 = 4$

$\Rightarrow \quad |\mathbf{a} - \mathbf{b}| = \sqrt{3}$

130. *(c)* Projection of $\mathbf{a}$ on $\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

$$= \frac{(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})}{|2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}|}$$

$$= \frac{6 - 3 + 5}{\sqrt{4 + 9 + 1}} = \frac{8}{\sqrt{14}}$$

131. *(a)* Let $f^{-1}(8) = x$

$\Rightarrow \quad f(x) = 8$

$\Rightarrow \quad x^3 = 8$

$\Rightarrow \quad x = 2 \qquad [\because x \in R]$

$\therefore \quad f^{-1}(8) = \{2\}$

132. *(d)* (a) When $x = -4$, then

$$4(-4) + 3y = 20 \Rightarrow y = 12$$

But $x = -4 \notin N$

$\therefore \quad (-4, 12) \notin R$

(b) When $x = 5$, then

$$4(5) + 3y = 20 \Rightarrow y = 0$$

$\therefore \quad (5, 0) \notin R$

(c) When $x = 3$, then

$$4(3) + 3y = 20 \Rightarrow y = \frac{8}{3}$$

$\therefore \quad (3, 4) \notin R$

(d) When $x = 2$, then

$$4(2) + 3y = 20 \Rightarrow y = 4$$

$\therefore \quad (2, 4) \in R$

133. *(b)* We have,

$$\log_{10} 7 = 0.8451$$

Let $x = 7^{-20}$

$\Rightarrow \log x = -20 \log 7 = -20 \times 0.8451 = -16.902$

$\Rightarrow x = 10^{-16.902}$

Now, -16.902 lies between -16 and -17

So, position of first significant figure of 7^{-20} is 17.

134. *(c)* We have,

$$= \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \frac{1}{8\cdot11} + \ldots + \frac{1}{[(2+(n-1)3)][5+(n-1)3]}$$

$$= \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \frac{1}{8\cdot11} + \ldots + \frac{1}{(3n-1)(3n+2)}$$

$$= \frac{1}{3}\left[\frac{3}{2\cdot5} + \frac{3}{5\cdot8} + \frac{3}{8\cdot11} + \ldots + \frac{3}{(3n-1)(3n+2)}\right]$$

$$= \frac{1}{3}\left[\frac{5-2}{2\cdot5} + \frac{8-5}{5\cdot8} + \frac{11-8}{8\cdot11} + \ldots + \frac{(3n+2)-(3n-1)}{(3n-1)(3n+2)}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \ldots + \frac{1}{3n-1} - \frac{1}{3n+2}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right] = \frac{1}{3}\left[\frac{3n+2-2}{2(3n+2)}\right] = \frac{n}{6n+4}$$

135. *(b)* We have,

$1! = 1, 4! = 24, 7! = 5040, 10! = 3628800,$
$11! = 39916800$

So, last two digit from 10! is always 00.

Now, $1! + 4! + 7! = 1 + 24 + 5040 = 5065$

So, ten's digit in given addition is 6 which is divisible by 3!.

136. *(b)* We have,

$$\frac{x^2}{2-\lambda} - \frac{y^2}{\lambda-5} - 1 = 0$$

$$\Rightarrow \quad \frac{x^2}{2-\lambda} + \frac{y^2}{5-\lambda} = 1$$

It will represent an ellipse, when

$$2 - \lambda > 0 \text{ and } 5 - \lambda > 0$$

$$\Rightarrow \quad \lambda < 2 \text{ and } \lambda < 5$$

$$\Rightarrow \quad \lambda < 2$$

137. *(d)* We have,

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Since, $(-4, 0)$ is vertex of the hyperbola, so equation of normal at $(-4, 0)$ is $y = 0$.

138. *(d)* We have,

$$p \rightarrow \sim q$$

The contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

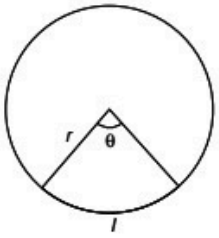
and converse of $p \rightarrow q$ is $q \rightarrow p$.

$\therefore$ Contrapositive of $p \rightarrow \sim q$ is $\sim(\sim q) \rightarrow \sim p \equiv q \rightarrow \sim p$.

Converse of $q \rightarrow \sim p$ is $\sim p \rightarrow q$.

139. *(a)* It is given that,

Perimeter of sector = Length of arc of the semi-circles



$$\Rightarrow \quad 2r + l = \pi r$$

$$\Rightarrow \quad 2r + r\theta = \pi r \qquad [\because l = r\theta]$$

$$\Rightarrow \quad r(2 + \theta) = \pi r$$

$$\Rightarrow \quad \theta = \pi - 2$$

140. *(c)* We have,

$$\tan 67\frac{1}{2}^\circ + \cot 67\frac{1}{2}^\circ$$

$$= \frac{\sin 67\frac{1}{2}^\circ}{\cos 67\frac{1}{2}^\circ} + \frac{\cos 67\frac{1}{2}^\circ}{\sin 67\frac{1}{2}^\circ}$$

$$= \frac{\sin^2 67\frac{1}{2}^\circ + \cos^2 67\frac{1}{2}^\circ}{\cos 67\frac{1}{2}^\circ \sin 67\frac{1}{2}^\circ}$$

$$= \frac{2}{2\sin 67\frac{1}{2}^\circ \cos 67\frac{1}{2}^\circ}$$

$$= \frac{2}{\sin 135^\circ}$$

$$= \frac{2}{\sin(90^\circ + 45^\circ)}$$

$$= \frac{2}{\cos 45^\circ} = 2 \times \sqrt{2} = 2\sqrt{2}$$

141. *(b)* We have, equation of hyperbola

$$3x^2 - 2y^2 = 25$$

$$\Rightarrow \quad \frac{x^2}{\left(\frac{5}{\sqrt{3}}\right)^2} - \frac{y^2}{\left(\frac{5}{\sqrt{2}}\right)^2} = 1$$

Equation of conjugate hyperbola is

$$\frac{y^2}{\left(\frac{5}{\sqrt{2}}\right)^2} - \frac{x^2}{\left(\frac{5}{\sqrt{3}}\right)^2} = 1$$

Now, eccentricity of hyperbola,

$$e_1 = \sqrt{1 + \frac{\left(\frac{5}{\sqrt{2}}\right)^2}{\left(\frac{5}{\sqrt{3}}\right)^2}}$$

$$= \sqrt{1 + \frac{3}{2}} = \sqrt{\frac{5}{2}}$$

and eccentricity of conjugate hyperbola,

$$e_2 = \sqrt{1 + \frac{\left(\frac{5}{\sqrt{3}}\right)^2}{\left(\frac{5}{\sqrt{2}}\right)^2}} = \sqrt{1 + \frac{2}{3}} = \sqrt{\frac{5}{3}}$$

Now, $e_1^2 + e_2^2 = \frac{5}{2} + \frac{5}{3} = \frac{25}{6} \simeq 4$

142. *(d)* We have,

$$p^2 - 2q^2 = 1$$

Let $p = 3, q = 2$

Then, $\quad p^2 - 2q^2$

$$= (3)^2 - 2(2)^2 = 9 - 8 = 1$$

$\therefore \quad p^2 - 2q^2 = 1$

Now, $p^2 + 2q^2 = (3)^2 + 2(2)^2$

$$= 9 + 8 = 17$$

143. *(b)* We have,

$$A(\text{adj}A) = 5I$$

$\Rightarrow \quad |A(\text{adj}A)| = |5I|$

$\Rightarrow \quad |A||\text{adj}A| = 5^3|I| \qquad [\because |KB| = K^n|B|]$

$\Rightarrow \quad |A||A|^{3-1} = 125 \times I \qquad [\because |\text{adj}A| = |A|^{n-1}]$

$\Rightarrow \quad |A|^3 = 125$

$\Rightarrow \quad |A| = 5$

Now, $|\text{adj}A| = |A|^{n-1}$

$$= |A|^2 \qquad [\because n = 3]$$

$$= (5)^2 = 25$$

144. *(a)* We have,

$$\sin 2x + \cos 4x = 2$$

$\Rightarrow \quad \sin 2x + 1 - 2\sin^2 2x = 2$

$\Rightarrow \quad 2\sin^2 2x - \sin 2x + 1 = 0$

Now, Discriminant $= b^2 - 4ac$

$$= (-1)^2 - 4(2)(1)$$

$$= 1 - 8 = -7$$

$$< 0$$

So, there is no real root of the given equation.

145. *(c)* Let $I = \int_{\text{II}} e^x \cdot \underset{\text{I}}{x^5} dx$

$$= x^5 \cdot e^x - \int e^x \cdot (5x^4) dx$$

$$= x^5 e^x - 5\int_{\text{II}} e^x \cdot \underset{\text{I}}{x^4} dx$$

$$= x^5 e^x - 5[x^4 \cdot e^x - \int e^x \cdot 4x^3 dx]$$

$$= x^5 e^x - 5x^4 e^x + 20\int_{\text{II}} e^x \cdot \underset{\text{I}}{x^3} dx$$

$$= x^5 e^x - 5x^4 e^x + 20[x^3 \cdot e^x - \int e^x \cdot 3x^2 dx]$$

$$= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60\int_{\text{II}} e^x \cdot \underset{\text{I}}{x^2} dx$$

$$= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60[x^2 \cdot e^x - \int e^x \cdot 2x dx]$$

$$= x^5 e^4 - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120\int e^x \cdot x dx$$

$$= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120 \ [xe^x - \int e^x dx]$$

$$= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120xe^x - 120e^x + C$$

$$= e^x[x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120] + C$$

146. *(b)* We have, $f(x)$ is an even function.

$\therefore \quad f(-x) = f(x)$

$\Rightarrow \quad f'(-x)(-1) = f'(x)$

$\Rightarrow \quad f'(x) + f'(-x) = 0$

$\Rightarrow \quad f'(e) + f'(-e) = 0$

147. *(a)* We have, $\alpha^2 + \alpha + 1 = 0$

$\Rightarrow \quad \alpha = \omega \text{ or } \omega^2$

Now, if $\alpha = \omega$, then

$$\alpha^{31} = \omega^{31} = (\omega^3)^{10} \cdot \omega = \omega = \alpha$$

and if $\alpha = \omega^2$, then

$$\alpha^{31} = (\omega^2)^{31} = \omega^{62} = (\omega^3)^{20} \cdot \omega^2 = \omega^2 = \alpha$$

$\therefore \quad \alpha^{31} = \alpha$

148. *(c)* Let $u = \sin(x^3)$

and $v = \cos(x^3)$

Then, $\dfrac{du}{dx} = \cos(x^3) \cdot (3x^2)$

and $\dfrac{dv}{dx} = -\sin(x^3)(3x^2)$

Now, $\dfrac{du}{dv} = \dfrac{\left(\dfrac{du}{dx}\right)}{\left(\dfrac{dv}{dx}\right)}$

$$= \frac{3x^2\cos(x^3)}{-3x^2\sin(x^3)} = -\cot(x^3)$$

149. *(d)* Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{b} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Then, unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ is $\dfrac{\mathbf{a} \times \mathbf{b}}{|\mathbf{a} \times \mathbf{b}|}$

Now, $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

and $|\mathbf{a} \times \mathbf{b}| = \sqrt{(1)^2 + (-1)^2 + (1)^2} = \sqrt{3}$

Thus, required vector $= \dfrac{\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}}{\sqrt{3}}$

150. *(b)* We have,

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \qquad [\because |A| = |A^T|]$$

$$= (-)\begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \qquad [R_1 \leftrightarrow R_2]$$

$$= (-)(-)\begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_2 \end{vmatrix} = B \quad [R_1 \leftrightarrow R_3]$$

151. *(c)* The locus of a point which moves such that the sum of its distances from the two fixed points is a constant is an ellipse.

152. *(a)* Centroid of $\Delta ABC = \left(\dfrac{2+8+5}{3}, \dfrac{3+10+5}{3}\right) = (5, 6)$

153. *(d)* If $3x^2 + xy - y^2 - 3x + 6y + k = 0$ represents a pair of lines, then

$$\begin{vmatrix} 3 & \frac{1}{2} & \frac{-3}{2} \\ \frac{1}{2} & -1 & 3 \\ \frac{-3}{2} & 3 & k \end{vmatrix} = 0$$

$$\Rightarrow 3(-k-9) - \frac{1}{2}\left(\frac{1}{2}k + \frac{9}{2}\right) - \frac{3}{2}\left(\frac{3}{2} - \frac{3}{2}\right) = 0$$

$$\Rightarrow \quad -3k - 27 - \frac{1}{4}k - \frac{9}{4} = 0$$

$$\Rightarrow \quad -\frac{13}{4}k - \frac{117}{4} = 0$$

$$k = -9$$

154. *(b)* (a) Put $x = 2, y = 2$, we have

$$(2)^2 + (2)^2 + 5 \times 2 + 5 \times 2 + 12$$

$$= 4 + 4 + 10 + 10 + 12 = 40 \neq 0$$

(b) Put $x = 2, \ y = 2$, we have

$$(2)^2 + (2)^2 - 5 \times 2 - 5 \times 2 + 12$$

$$= 4 + 4 - 10 - 10 + 12 = 0$$

Put $x = 3, y = 3$, we get

$$(3)^2 + (3)^2 - 5 \times 3 - 5 \times 3 + 12$$

$$= 9 + 9 - 15 - 15 - 12 = 30 - 30 = 0$$

So, both points satisfies the given equation.

(c) Put $x = 2, y = 2$, we get

$$(2)^2 + (2)^2 + 5 \times 2 - 5 \times 2 + 12$$

$$= 4 + 4 + 10 - 10 + 12 = 20 \neq 0$$

(d) Put $x = 2, y = 2$, we get

$$(2)^2 + (2)^2 - 5 \times 2 + 5 \times 2 - 12$$

$$= 4 + 4 - 10 + 10 - 12 = -4 \neq 0$$

155. *(a)* We have,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

which is a lower triangular matrix.

Since, characteristic roots of a lower triangular matrix are its diagonal elements.

So, charateristics roots of A are 1, 3, 6.

156. *(a)* If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$A^{-1}=\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

So, if $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then

$$A^{-1}=\frac{1}{4-6}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$=-\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

157. *(d)* We have,

$A=\{-1,0,1\}$

Now, let e be the identity element.

Then, $a\times e=a\Rightarrow e=1\in A$

Again, a^{-1} be the inverse element.

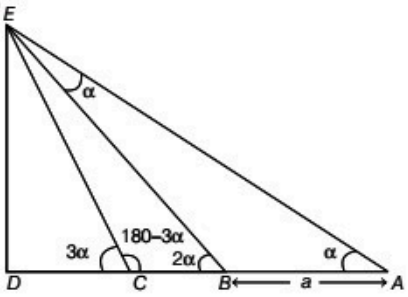
Then, $a\times a^{-1}=e$

$\Rightarrow$ $a\times a^{-1}=1\Rightarrow a^{-1}=\frac{1}{a}$

Now, for $a=0$, $a^{-1}=\frac{1}{0}=\infty\notin A$

So, inverse law fails.

158. *(c)*



Let DE be the height of the tower.

Now, in ΔABE,

$\angle AEB=\angle EBC-\angle EAB$

$=2\alpha-\alpha=\alpha$

$\therefore$ $\angle AEB=\angle BAE$

$\Rightarrow$ $BE=AB=a$

Now, $\angle ECB=180°-\angle DCE=180-3\alpha$

Now, in ΔBEC, $\frac{BE}{\sin(180-3\alpha)}=\frac{CE}{\sin 2\alpha}$ [using sine rule]

$\Rightarrow$ $\frac{a}{\sin 3\alpha}=\frac{CE}{\sin 2\alpha}$

$\Rightarrow$ $CE=\frac{a\sin 2\alpha}{\sin 3\alpha}$

In ΔEDC,

$\sin 3\alpha=\frac{DE}{CE}$

$\Rightarrow$ $DE=CE\sin 3\alpha$

$=\frac{a\sin 2\alpha}{\sin 3\alpha}\times\sin 3\alpha=a\sin 2\alpha$

Hence, height of tower is $a\sin 2\alpha$.

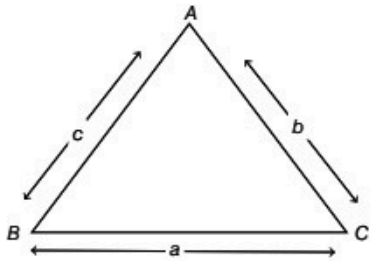
159. *(c)* It is given that A, B, C are in AP.

$\therefore$ $2B=A+C$

Now, $A+B+C=180°$

$\Rightarrow$ $B+2B=180°$

$\Rightarrow$ $B=60°$



Now, using sine rule, we have

$\frac{b}{\sin B}=\frac{c}{\sin C}$

$\Rightarrow$ $\sin C=\frac{c}{b}\sin B$

$=\frac{\sqrt{2}}{\sqrt{3}}\times\sin 60°$ $[\because b:c=\sqrt{3}:\sqrt{2}]$

$=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{2}=\frac{1}{\sqrt{2}}$

$\therefore$ $c=45°$

Now, $A+B+C=180°$

$\Rightarrow$ $A+60°+45°=180°$

$\Rightarrow$ $A=75°$

160. *(a)* We have,

$$\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right)$$

$$=\sin\left\{\sin^{-1}\left(2\sqrt{\frac{63}{65}}\sqrt{1-\left(\sqrt{\frac{63}{65}}\right)^2}\right)\right\}$$

$[\because 2\sin^{-1}x=\sin^{-1}2x\sqrt{1-x^2}]$

$$= \sin\left\{\sin^{-1}\left(2\sqrt{\frac{63}{65}}\sqrt{1-\frac{63}{65}}\right)\right\}$$

$$= \sin\left\{\sin^{-1}\left(2\sqrt{\frac{63}{65}}\times\sqrt{\frac{2}{65}}\right)\right\} = \sin\left\{\sin^{-1}\frac{2\sqrt{126}}{65}\right\}$$

$$= \frac{2\sqrt{126}}{65} \qquad [\because \sin(\sin^{-1}x) = x]$$

161. *(c)* We have, $|\sin x| = \cos x$

$\Rightarrow \sin x = \pm\cos x$

$\Rightarrow \tan x = \pm 1$

$\Rightarrow \tan x = \tan\left(\pm\frac{\pi}{4}\right)$

$\Rightarrow x = n\pi \pm \frac{\pi}{4}$

162. *(d)* We have,

$$x^3 - 6x + 9 = 0$$

$\Rightarrow (x+3)(x^2-3x+3) = 0$

$\Rightarrow x + 3 = 0 \qquad [\because x^2 - 3x + 3 \neq 0, x \in R]$

$\Rightarrow x = -3$

163. *(d)* We know that,

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$ and so on.

So, unit place is always 5.

So, unit's place of 5^{834} is 5.

164. *(d)* We have,

$3^2 = 4 \pmod 5$

$(3^2)^2 = 16 \pmod 5 = 1 \pmod 5$

$(3^4)^{25} = (1)^{25} \pmod 5$

$\Rightarrow 3^{100} = 1 \pmod 5$

Again, $2^2 = 4 \pmod 5$

$(2^2)^2 = 16 \pmod 5 = 1 \pmod 5$

$(2^4)^{12} = (1)^{12} \pmod 5 = 1 \pmod 5$

$2^{48} \cdot 2^2 = 4 \pmod 5$

$\therefore 3^{100} \times 2^{50} = 1 \times 4 \pmod 5 = 4 \pmod 5$

165. *(a)* Let $I = \int \frac{\sin x \cos x}{\sqrt{1-\sin^4 x}}dx$

Put $\sin^2 x = t \Rightarrow 2\sin x \cos x\, dx = dt$

$\therefore \quad I = \frac{1}{2}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{2}\sin^{-1} t + C \quad = \frac{1}{2}\sin^{-1}(\sin^2 x) + C$

166. *(b)* Let $I = \int_{-2}^{2} (ax^3 + bx + c)dx$ ...(i)

$\Rightarrow I = \int_{-2}^{2}[(a(-2+2-x)^3 + b(-2+2-x) + c)]dx$

$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

$= \int_{-2}^{2} (-ax^3 - bx + c)dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2I = \int_{-2}^{2} 2c\, dx$

$I = c(x)_{-2}^{2} = 4c$

So, I depends on value of c.

167. *(b)* We have,

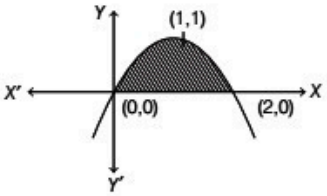
$y = 2x - x^2$

$\Rightarrow y = -(x^2 - 2x)$

$\Rightarrow y = -[(x-1)^2 - 1]$

$\Rightarrow y = -(x-1)^2 + 1$

$\Rightarrow (x-1)^2 = -(y-1)$



Required area $= \int_0^2 y\, dx$

$= \int_0^2 (2x - x^2)dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3}$

$= \frac{4}{3}$ sq unit

168. *(d)* We have,

$y\frac{dy}{dx} + x = c$

$\Rightarrow y\frac{dy}{dx} = (c - x)$

$\Rightarrow y\, dy = (c - x)dx$

$$\Rightarrow \quad y\,dy = -(x-c)dx$$

$$\Rightarrow \quad \int y\,dy = -\int (x-c)dx$$

$$\Rightarrow \quad \frac{y^2}{2} = -\frac{(x-c)^2}{2} + K$$

$$\Rightarrow \quad (x-c)^2 + y^2 = 2K$$

which is a circle with centre $(c, 0)$ on the X-axis.

169. *(a)* We have,

$$f(x^5) = 5x^3$$

$$\Rightarrow \quad f(x) = 5x^{3/5}$$

$$\Rightarrow \quad f'(x) = 5 \times \frac{3}{5} x^{\frac{3}{5}-1} = 3x^{-\frac{2}{5}}$$

$$= \frac{3}{\sqrt[5]{x^2}}$$

170. *(d)* We have,

$$f(x) = \begin{cases} 2a - x, & -a < x < a \\ 3x - 2a, & x \geq a \end{cases}$$

At $x = a$

$$\text{LHL} = \lim_{x \to a}(2a - x) = 2a - a = a$$

$$\text{RHL} = \lim_{x \to a}(3x - 2a) = 3a - 2a = a$$

$$f(a) = 3a - 2a = a$$

$\because$ LHL = RHL = $f(a)$.

So, $f(x)$ is continuous at $x = a$.

Now, when $x < a$, $f(x) = 2a - x$,

which is continuous for all $x < a$.

Again, when $x > a$, $f(x) = 3x - 2a$, which is continuous for all $x > a$.

So, $f(x)$ is continuous for all x.

Now, at $x = a$

$$\text{LHD} = \lim_{x \to a} \frac{(2a - x) - a}{x - a}$$

$$= \lim_{x \to a} \frac{-(x-a)}{(x-a)} = -1$$

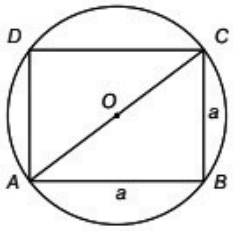
$$\text{and RHD} = \lim_{x \to a} \frac{(3x - 2a) - a}{x - a}$$

$$= \lim_{x \to a} \frac{3(x-a)}{x-a} = 3$$

$\therefore$ LHD $\neq$ RHD

So, $f(x)$ is not differentiable at $x = a$.

171. *(c)* The area of a rectangle that can be inscribed in a circle will have maximum area when it is a square.



Let a be the side of square $ABCD$.

Then, $\quad AB^2 + BC^2 = AC^2$

$$\Rightarrow \quad a^2 + a^2 = (4)^2$$

$$\Rightarrow \quad 2a^2 = 16$$

$$\Rightarrow \quad a^2 = 8$$

$\therefore$ Area of square is 8 sq units.

172. *(b)* Let $Z = x + iy$

Then, $\overline{Z} = x - iy$

Now, it is given that

$$Z = -\overline{Z}$$

$$\Rightarrow \quad x + iy = -(x - iy)$$

$$\Rightarrow \quad x + iy = -x + iy$$

$$\Rightarrow \quad 2x = 0$$

$$\Rightarrow \quad x = 0$$

$\therefore$ Z is purely imaginary.

173. *(a)* We have,

$$\sum_{K=1}^{6} \left(\sin \frac{2K\pi}{7} - i \cos \frac{2K\pi}{7} \right)$$

$$= -i \sum_{K=1}^{6} \left(\cos \frac{2K\pi}{7} + i \sin \frac{2K\pi}{7} \right)$$

$$= -i \sum_{K=1}^{6} e^{\frac{2K\pi}{7}}$$

$$= -i \left[e^{\frac{2\pi}{7}} + e^{\frac{4\pi}{7}} + e^{\frac{6\pi}{7}} + e^{\frac{8\pi}{7}} + e^{\frac{10\pi}{7}} + e^{\frac{12\pi}{7}} \right]$$

$$= -i \left[1 + e^{\frac{2\pi}{7}} + e^{\frac{4\pi}{7}} + e^{\frac{6\pi}{7}} + e^{\frac{8\pi}{7}} + e^{\frac{10\pi}{7}} + e^{\frac{12\pi}{7}} - 1 \right]$$

$= -i[0 - 1]$ [$\because$ Sum of all nth roots of unit is zero]

$= i$

174. *(c)* We have,

$$\lim_{x\to\infty} x\sin\left(\frac{2}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)} \times 2$$

Put $\frac{2}{x} = y$.

Then $x \to \infty$

$\Rightarrow \frac{2}{x} \to 0 \Rightarrow y \to 0$

$$\therefore \lim_{x\to\infty} \frac{\sin\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)} \times 2$$

$$= \lim_{y\to 0} \frac{\sin y}{y} \times 2 = 1 \times 2 = 2$$

175. *(b)* We have,

$$x = 80t - 16t^2$$

$$\Rightarrow \quad \frac{dx}{dt} = 80 - 32t$$

For maximum value of x,

$$\frac{dx}{dt} = 0$$

$$\Rightarrow 80 - 32t = 0 \Rightarrow \quad t = \frac{80}{32} = 2.5$$

Again, $\frac{d^2x}{dt^2} = -32 < 0$

So, at $t = 2.5$ sec, x is maximum.

176. *(d)* We have,

$$f(x) = \frac{\log x}{x}$$

$$\Rightarrow f'(x) = \frac{\frac{1}{x} \times x - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$$

For maxima or minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{1 - \log x}{x^2} = 0$$

$$\Rightarrow \quad \log x = 1$$

$$\Rightarrow \quad x = e$$

Now, $f''(x) = \frac{-\frac{1}{x} \cdot x^2 - (1 - \log x) \cdot 2x}{x^4}$

$$= \frac{-x - 2x + 2x\log x}{x^4}$$

$$= \frac{-3x + 2x\log x}{x^4} = \frac{-3 + 2\log x}{x^3}$$

$$\therefore \quad f''(e) = \frac{-3+2}{e^3} = \frac{-1}{e^3} < 0$$

Hence, $f(x)$ attains a local maxima at $x = e$.

$$\therefore \quad f_{\max} = f(e) = \frac{\log e}{e} = \frac{1}{e}$$

177. *(c)* We have,

$$f(x) = be^{ax} + ae^{bx}$$

$$\Rightarrow \quad f'(x) = abe^{ax} + abe^{bx}$$

$$\Rightarrow \quad f''(x) = a^2be^{ax} + ab^2e^{bx}$$

$$\therefore \quad f''(0) = a^2b + ab^2 = ab(a+b)$$

178. *(c)* We have,

$$\sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{x}{y}$$

$$\Rightarrow \quad \sqrt{\frac{2\cos^2\frac{A}{2}}{2\sin^2\frac{A}{2}}} = \frac{x}{y}$$

$$\Rightarrow \quad \cot\frac{A}{2} = \frac{x}{y} \Rightarrow \tan\frac{A}{2} = \frac{y}{x}$$

Now, $\tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}} = \frac{2\left(\frac{y}{x}\right)}{1-\left(\frac{y}{x}\right)^2} = \frac{2xy}{x^2-y^2}$

179. *(a)* Let $I = \int \frac{\sec x}{\sec x + \tan x} dx$

$$= \int \frac{1}{1+\sin x} dx = \int \frac{1-\sin x}{1-\sin^2 x} dx$$

$$= \int \frac{1-\sin x}{\cos^2 x} dx$$

$$= \int (\sec^2 x - \sec x \tan x) dx$$

$$= \tan x - \sec x + C$$

180. *(b)* We have,

$$\int f(x)dx = g(x)$$

$$\Rightarrow \quad f(x) = g'(x)$$

Now, let $\quad I = \int f(x) \cdot g(x) dx = \int g'(x) g(x) dx$

Put $g(x) = t$

$$\Rightarrow \quad g'(x)dx = dt$$

$$\therefore \quad I = \int t\, dt = \frac{t^2}{2} + C = \frac{1}{2}(g(x))^2 + C$$