SOLVED PAPER – 2020 (COMEDK)

Instructions

- There are 180 questions in all. The number of questions in each section is as given below.

Sections	No. of Questions
Section I : Physics	1–60
Section II : Chemistry	61–120
Section III : Mathematics	121–180

- All the questions are Multiple Choice Questions having four options out of which **ONLY ONE** is correct.
- Candidates will be awarded 1 mark for each correct answer. There will be no negative marking for incorrect answer.
- Time allotted to complete this paper is 3 hrs.

PHYSICS

1. Light of frequency 10^{15} Hz falls on a metal surface of work function 2.5 eV. The stopping potential of photoelectrons (in V) is

a. 1.6 ***b.*** 2.5 ***c.*** 4.1 ***d.*** 6.6

2. A proton accelerated through a potential V has de-Broglie wavelength λ. Then, the de-Broglie wavelength of an α-particle, when accelerated through the same potential V is

a. $\frac{\lambda}{2}$ ***b.*** $\frac{\lambda}{\sqrt{2}}$ ***c.*** $\frac{\lambda}{2\sqrt{2}}$ ***d.*** $\frac{\lambda}{8}$

3. Consider a thin spherical shell of radius R consisting of uniform surface charge density s. The electric field at a point of distance x from its centre and outside the shell is

a. inversely proportional to s
b. directly proportional to x^2
c. directly proportional to R
d. inversely proportional to x^2

4. An electron of an atom transits from n_1 to n_2. In which of the following maximum frequency of photon will be emitted?

a. $n_1 = 1$ to $n_2 = 2$
b. $n_1 = 2$ to $n_2 = 1$
c. $n_1 = 2$ to $n_2 = 6$
d. $n_1 = 6$ to $n_2 = 2$

5. Two protons are kept at a separation of 40 Å. F_n is the nuclear force and F_e is the electrostatic force between them. Then,

a. $F_n << F_e$
b. $F_n = F_e$
c. $F_n >> F_e$
d. $F_n = F_e$

6. Two radioactive materials X_1 and X_2 have decay constant 5λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of X_1 to that X_2 will be $1/e$ after a time

a. $1/4\lambda$ ***b.*** e/λ
c. λ ***d.*** $\frac{1}{2}\lambda$

7. The Poisson's ratio of a material is 0.1. If the longitudinal strain of a rod of this material is 10^{-3}, then the percentage change in the volume of the rod will be

a. 0.008% ***b.*** 0.08%
c. 0.8% ***d.*** 8%

8. A satellite can be in a geostationary orbit around a planet if it is at a distance R from the centre of the planet. If the planet starts rotating about its axis with double the angular velocity, then to make the satellite geostationary, its orbital radius should be

a. $2R$ *b.* $\frac{R}{2}$ *c.* $\frac{R}{2^{1/3}}$ *d.* $\frac{R}{4^{1/3}}$

9. When a *p-n* junction diode is connected in forward bias, its barrier potential

a. decreases and less current flows in the circuit
b. decreases and more current flows in the circuit
c. increases and more current flows in the circuit
d. decreases and no current flows in the circuit

10. Ground waves have wavelength

a. less than that of sky waves
b. greater than that of sky waves
c. less than that of space waves
d. equal to that of space waves

11. A ball floats on the surface of water in a container exposed to the atmosphere. When the container is covered and the air is partially removed, then the ball

a. rises
b. gets immersed more in water
c. remains immersed at its former depth
d. may rise or sink

12. A frame made of metallic wire enclosing a surface area A is covered with a soap film. If the area of the frame of metallic wire is reduced by 50%, then the energy of the soap film will be changed by

a. 100% *b.* 75%
c. 50% *d.* 25%

13. A compound slab is made of two parallel plates of copper and brass of the same thickness and having thermal conductivities in the ratio 4 : 1. The free face of copper is at 0°C. The temperature of the interface is 20°C. What is the temperature of the free face of brass?

a. 0°C *b.* 20°C
c. 40°C *d.* 100°C

14. 1 mm^3 of a gas is compressed at 1 atmospheric pressure and temperature 27°C to 627°C. What is the final pressure under adiabatic condition? (γ for the gas = 1.5)

a. 27×10^5 N/m^2 *b.* 80×10^5 N/m^2
c. 36×10^5 N/m^2 *d.* 56×10^5 N/m^2

15. If sink is at a temperature of – 39°C and source at 0°C, then efficiency will be

a. 39.4% *b.* 14.2%
c. 35.2% *d.* 45.5%

16. Which of the following laws of Physics is valid across all domains of nature?

a. Newton's law of motion
b. Conservation of momentum
c. Conservation of energy
d. All of the above

17. A particle of mass m is moving in a horizontal circle of radius R under a centripetal force equal to $-\frac{A}{R^2}$ (A = constant). The total energy of the particle is

a. $\frac{A}{R}$ *b.* $-\frac{A}{R}$
c. $\frac{A}{2R}$ *d.* $-\frac{A}{2R}$

18. A force of 20 N is applied on a body of mass 5 kg resting on a horizontal plane. The body gains a kinetic energy of 10 J after it moves a distance 2 m. The frictional force is

a. 10 N *b.* 15 N
c. 20 N *d.* 30 N

19. A body under the action of a force $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$, acquires an acceleration of 1 m/s^2. The mass of this body must be

a. 10 kg *b.* 20 kg
c. $10\sqrt{2}$ kg *d.* $2\sqrt{10}$ kg

20. A body of mass 1000 kg is moving horizontally with a velocity 50 m/s. A mass of 250 kg is added. Find the final velocity.

a. 40 m/s *b.* 20 m/s
c. $30\sqrt{2}$ m/s *d.* 50 m/s

21. Equal volumes of two gases, having their densities in the ratio of 1 : 16 exert equal pressures on the walls of two containers. The ratio of their rms velocities $\left(\frac{c_1}{c_2}\right)$

a. 1 : 8 *b.* 8 : 1
c. 1 : 4 *d.* 4 : 1

22. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio C_p / C_V of the mixture is

a. 1.4 *b.* 1.54
c. 1.59 *d.* 1.62

23. A particle executes a linear SHM with an amplitude of 4 cm. At the mean position the velocity of the particle is 10 cm/s. What is the displacement of the particle when its speed becomes 5 cm/s?

a. $2(\sqrt{3})$ cm *b.* $2(\sqrt{5})$ cm
c. $\sqrt{5}$ cm *d.* $\sqrt{3}$ cm

24. The equation of a progressive wave can be given by $y = 15\sin(660\pi t - 0.02\pi x)$ cm. The frequency of the wave is

a. 330 Hz *b.* 342 Hz
c. 365 Hz *d.* 660 Hz

25. A source of sound gives 5 beats per second, when sounded with another source of frequency 100 s^{-1}. The second harmonic of the source, together with a source of frequency 205 s^{-1} gives 5 beats per second. What is frequency of the source?

a. 95 s^{-1} *b.* 105 s^{-1}
c. 100 s^{-1} *d.* 205 s^{-1}

26. A charge of 0.8 C is divided into two charges Q_1 and Q_2. These are kept at a separation of 30 cm. The force on Q_1 is maximum when

a. $Q_1 = Q_2 = 0.4$ C
b. $Q_1 = 0.8$ C, Q_2 negligible
c. Q_1 negligible, $Q_2 = 0.8$ C
d. $Q_1 = 0.2$ C, $Q_2 = 0.6$ C

27. An electric dipole has a pair of equal and opposite point charges q and $-q$ separated by a distance $2x$. The axis of the dipole is defined as

a. direction from positive charge to negative charge
b. direction from negative charge to positive charge
c. perpendicular to the line joining the two charges drawn at the centre and pointing upward direction
d. perpendicular to the line joining the two charges at the centre and pointing downward direction

28. An electric dipole is placed in a uniform electric field with the dipole axis making an angle θ with the direction of the electric field. The orientation of the dipole for stable equilibrium is

a. $\frac{\pi}{6}$ *b.* $\frac{\pi}{3}$ *c.* 0 *d.* $\frac{\pi}{2}$

29. Two point charges $A = +3$ nC and $B = +1$ nC are placed 5 cm apart in air. The work done to move charge B towards A by 1 cm is

a. 2.0×10^{-7} J *b.* 1.35×10^{-7} J
c. 2.7×10^{-7} J *d.* 121×10^{-7} J

30. The potential energies associated with four orientations of an electric dipole in an electric field are

(i) $-5U_0$ (ii) $-7U_0$
(iii) $3U_0$ (iv) $5U_0$

where U_0 is positive. Rank the orientations according to the angle between the electric dipole moment **p** and electric field **E**, greatest first

a. (i), (ii), (iii), (iv) *b.* (ii), (iii), (i), (iv)
c. (iv), (iii), (i), (ii) *d.* (iv), (i), (iii), (ii)

31. Suppose refractive index μ is given as $\mu = A + \frac{B}{\lambda^2}$, where A and B are constants and λ is wavelength, then dimensions of B are same as that of

a. wavelength *b.* volume
c. pressure *d.* area

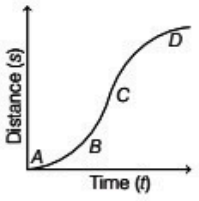
32. If voltage $V = (200 \pm 8)$ V and current $I = (20 \pm 05)$ A, then the percentage error in resistance R is

a. 5.2% *b.* 25% *c.* 3% *d.* 6.5%

33. A body is projected vertically upwards. The times corresponding to height h, while ascending and while descending are t_1 and t_2, respectively. Then, the velocity of projection is (g is acceleration due to gravity)

a. $\frac{g\sqrt{t_1 t_2}}{2}$ *b.* $\frac{g(t_1 + t_2)}{2}$
c. $g\sqrt{t_1 t_2}$ *d.* $\frac{g t_1 t_2}{t_1 + t_2}$

34. A particle shows distance-time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point



a. D *b.* A *c.* B *d.* C

35. A fluid is in streamline flow across a horizontal pipe of variable area of cross-section. For this which of the following statements is correct?

a. Velocity of fluid is maximum at narrowest part.
b. Pressure of the fluid is maximum at widest part.
c. Both (a) and (b)
d. Neither (a) nor (b)

36. Two capacitors C_1 and C_2 are charged to 120 V and 200 V, respectively. When they are connected in parallel, it is found that potential on each one of them is zero. Therefore,

a. $5C_1 = 3C_2$ *b.* $3C_1 = 5C_2$
c. $3C_1 + 5C_2 = 0$ *d.* $9C_1 = 4C_2$

37. Point out the right statements about the validity of Kirchhoff's junction rule,

a. it is based on conservation of charge.
b. outgoing currents add up and are equal to incoming currents at a junction.
c. bending or reorienting the wire does not change the validity of Kirchhoff's junction rule.
d. All of the above

38. Four particles each of the mass m are placed at the corners of a square of side length l. The radius of gyration of the system about an axis perpendicular to the square and passing through its centre is

a. $\frac{l}{\sqrt{2}}$ *b.* $\frac{l}{2}$ *c.* l *d.* $\sqrt{2}l$

39. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of Young's modulus of steel to that of copper?

a. 1.8 : 1 *b.* 2.8 : 1 *c.* 3.8 : 1 *d.* 4.8 : 1

40. The masses of 200 g and 300 g are attached to the 20 cm and 70 cm marks of a light meter rod, respectively. The moment of inertia of the system about an axis passing through 50 cm mark is

a. 0.15 kg m^2 *b.* 0.036 kg m^2
c. 0.3 kg m^2 *d.* Zero

41. Two spherical bodies of masses M and $5M$ and radii R and $2R$ are released in free space with initial separation between their centres equal to 12 R. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is

a. 2.5 R *b.* 4.5 R *c.* 7.5 R *d.* 1.5 R

42. If $P = Q = R = 10\ \Omega$ and $S = 20\ \Omega$, then what resistance should be joined with S to balance the Wheatstone's network?

a. Join a resistance of 10 Ω in series with S
b. Join a resistance of 10 Ω in parallel with S
c. Join a resistance of 20 Ω in parallel with S
d. Join a resistance of 20 Ω in series with S

43. To the potentiometer wire of length L and 10 Ω resistance, a battery of emf 2.5 V and a resistance R are connected in series. If a potential difference of 1 V is balanced across L / 2 length, the value of R in Ω will be

a. 1 *b.* 1.5
c. 2 *d.* 2.5

44. An electron moves with speed of 2×10^5 m/s along the positive x-direction in a magnetic field $B = (\hat{i} - 4\hat{j} - 3\hat{k})$ T. The magnitude of the force (in N) experienced by the electron is

a. 1.18×10^{-13} *b.* 1.28×10^{-13}
c. 1.6×10^{-13} *d.* 1.72×10^{-13}

45. A horizontal overhead power line carries a current of 90 A in East to West direction. What is the magnitude and direction of the magnetic field due to the current, 1.5 m below the line?

a. 1.2×10^{-5} T, towards South
b. 1.2×10^{-5} T, towards North
c. 1.2×10^{-5} T, towards East
d. 1.2×10^{-5} T, towards West

46. A moving coil galvanometer has 28 turns and area of coil is 4×10^{-2} m^2. If the magnetic field is 0.2 T, then to increase the sensitivity by 25% without changing area and field, the number of turns should be changed to

a. 24 *b.* 35
c. 60 *d.* 54

47. The intensity of magnetic field due to an isolated pole of strength m_p at a point distant r from it will be

a. m_p / r^2 *b.* $m_p r^2$
c. r^2 / m_p *d.* m_p / r

48. The particle that cannot be accelerated by a cyclotron is

a. proton *b.* α-particle
c. electron *d.* deuteron nucleus

49. The angle which the total magnetic field of earth makes with the surface of the earth is called

a. declination *b.* magnetic meridian
c. geographic meridian *d.* inclination

50. The angle of dip of at a place where horizontal and vertical components of earth's magnetic field are equal is

a. 45° *b.* 30° *c.* 0° *d.* 60°

51. A coil of wire of a certain radius has 100 turns and a self inductance of 15 mH. The self inductance of a second similar coil of 500 turns will be

a. 75 mH ***b.*** 375 mH
c. 15 mH ***d.*** None of the above

52. A coil of 100 turns carries a current of 5 mA and creates a magnetic flux of 10^{-5} Wb. The inductance is

a. 0.2 mH ***b.*** 2.0 mH
c. 0.02 mH ***d.*** None of these

53. In step-up transformer, relation between number of turns in primary (N_P) and number of turns in secondary (N_S) coils is

a. N_S is greater than N_P
b. N_P is greater than N_S
c. N_S is equal to N_P
d. $N_P = 2N_S$

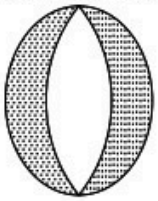
54. For a series *L-C-R* circuit at resonance, which statement is not true?

a. Wattless current is zero.
b. Power factor is zero.
c. Peak energy stored by a capacitor = peak energy stored by an inductor.
d. Average power = apparent power.

55. Which of the following has/have zero average value in a plane electromagnetic wave?

a. Electric field
b. Magnetic field
c. Both (a) and (c)
d. Magnetic energy

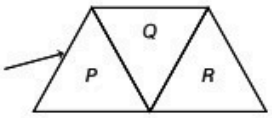
56. A convex lens is made of 3 layers of glass of 3 different materials as in the figure.



A point object is placed on its axis. The number of images of the object are

a. 3 ***b.*** 4
c. 1 ***d.*** 2

57. A ray of light suffers minimum deviation in equilateral prism *P*. Additional prisms *Q* and *R* of identical shape and of same material as that of *P* are now combined as shown in figure. The ray will now suffer



a. greater deviation
b. no deviation
c. same deviation as before
d. total internal reflection

58. Two identical light waves, propagating in the same direction, have a phase difference δ. After they superpose the intensity of the resulting wave will be proportional to

a. $\cos\delta$ ***b.*** $\cos(\delta/2)$
c. $\cos^2(\delta/2)$ ***d.*** $\cos^2\delta$

59. A plastic sheet (refractive index = 1.6) covers one slit of a double slit arrangement for the Young's experiment. When the double slit is illuminated by monochromatic light (wavelength = 5867 Å), the centre of the screen appears dark rather than bright. The minimum thickness of the plastic sheet to be used for this to happen is

a. 3300 Å ***b.*** 6600 Å
c. 2062 Å ***d.*** 5500 Å

60. A particle starts moving from point (2, 10, 1). Displacement for the particle is $8\hat{i} - 2\hat{j} + \hat{k}$. The final coordinates of the particle is

a. (10, 8, 2) ***b.*** (8, 10, 2)
c. (2, 10, 8) ***d.*** (8, 2, 10)

CHEMISTRY

61. During the extraction of gold the following reactions take place

$$Au + CN^- + H_2O \xrightarrow{O_2} [X]$$

$$[X] + Zn \longrightarrow [Y] + Au$$

X and Y are respectively

a. $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

b. $[Au(CN)_4]^{3-}$ and $[Zn(CN)_4]^{2-}$

c. $[Au(CN)_4]^{2-}$ and $[Zn(CN)_4]^{2-}$

d. $[Au(CN)_2]^-$ and $[Zn(CN)_6]^{4-}$

62. The number of gram molecules of chlorine in 6.02×10^{25} hydrogen chloride molecules is

a. 5 *b.* 50

c. 100 *d.* 10

63. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that, graphite

a. has molecules of variable molecular masses like polymers.

b. has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds.

c. is a non-crystalline substance.

d. is an allotropic form of carbon.

64. Paracetamol is a/an

a. antimalarial

b. antipyretic

c. analgesic

d. Both (b) and (c)

65. Which one of the following has maximum number of atoms of oxygen?

a. 2 g of water

b. 2 g of sulphur dioxide

c. 2 g of carbon dioxide

d. 2 g of carbon monoxide

66. Which one of the following shows functional isomerism?

a. CH_2Cl_2 *b.* C_2H_5OH

c. C_3H_6 *d.* C_2H_4

67. In the ionic equation,
$BiO_3^- + 6H^+ + xe^- \longrightarrow Bi^{3+} + 3H_2O$,
the value of x is

a. 3 *b.* 4

c. 2 *d.* 6

68. Molarity of a given orthophosphoric acid solution is 3M. It's normality is

a. 1 N *b.* 3 N

c. 0.3 N *d.* 9 N

69. Acidified sodium fusion extract on addition of ferric chloride solution gives blood red colouration which confirms the presence of

a. S *b.* N

c. N and S *d.* S and Cl

70. A body of mass 10 mg is moving with a velocity of 100 ms^{-1}. The wavelength of de-Broglie wave associated with it would be
($h = 6.63 \times 10^{-34}$ Js)

a. 6.63×10^{-37} m

b. 6.63×10^{-31} m

c. 6.63×10^{-34} m

d. 6.63×10^{-35} m

71. Mg^{2+} is isoelectronic with

a. Ca^{2+} *b.* Na^+

c. Zn^{2+} *d.* Cu^{2+}

72. Gram molecular volume of oxygen at STP is

a. 11200 cm^3 *b.* 22400 cm^3

c. 5600 cm^3 *d.* 3200 cm^3

73. Presence of halogen in organic compounds can be detected using

a. Beilstien's test *b.* Kjeldahl test

c. Duma's test *d.* Liebig's test

74. The electronic configuration of Cr^{3+} is

a. [Ar] $3d^5\,4s^1$ *b.* [Ar] $3d^2\,4s^1$

c. [Ar] $3d^3\,4s^0$ *d.* [Ar] $3d^4\,4s^2$

75. The mass of a metal, with equivalent mass 31.75, which would combine with 8 g of oxygen is

a. 31.75 *b.* 3.175 *c.* 8 *d.* 1

76. Benzene reacts with chlorine in sunlight to give a final product

a. C_6H_5Cl *b.* C_6Cl_6

c. $C_6H_6Cl_6$ *d.* CCl_4

77. In the periodic table metals usually used as catalysts belong to

a. *s*-block *b.* *p*-block

c. *d*-block *d.* *f*-block

78. Dalton's law of partial pressures is applicable to which one of the following systems?
a. $CO + H_2$ *b.* $H_2 + Cl_2$
c. $NO + O_2$ *d.* $NH_3 + HCl$

79. The general formula of a cycloalkene is
a. C_nH_{2n+2} *b.* C_nH_{2n-2}
c. C_nH_{2n} *d.* C_nH_n

80. In acetylene molecule, between the carbon atoms there are
a. three σ-bonds
b. two σ and one π-bonds
c. one σ and two π -bonds
d. three π-bonds

81. Denatured alcohol is
a. rectified spirit
b. undistilled ethanol
c. rectified spirit + methanol + naphtha
d. ethanol + methanol

82. During the formation of a chemical bond
a. energy decreases
b. energy increases
c. energy of the system does not change
d. electron-electron repulsion becomes more than the nucleus-electron attraction

83. One mole of oxygen at 273 K and one mole of sulphur dioxide at 546 K are taken in two separate containers, then
a. kinetic energy of O_2 > kinetic energy of SO_2
b. kinetic energy of O_2 < kinetic energy of SO_2
c. kinetic energy of both are equal
d. None of the above

84. + *I*-effect is shown by
a. $-NO_2$ *b.* —Cl
c. —Br *d.* $-CH_3$

85. Formation of coloured solution is possible, when metal ion in the compound contains
a. paired electrons
b. unpaired electrons
c. lone pair of electrons
d. None of the above

86. Which of the following is an intensive property?
a. Temperature
b. Surface tension
c. Viscosity
d. All of the above

87. Hofmann's bromamide reaction is to convert
a. amine to amide *b.* amide to amine
c. alcohol to acid *d.* acid to alcohol

88. IUPAC name of $Na_3[Co(NO_2)_6]$ is
a. sodium cobaltonitrite
b. sodium hexanitrito cobaltate (III)
c. sodium hexanitro cobaltate (III)
d. sodium hexanitrito cobaltate (II)

89. Thermodynamic standard conditions of temperature and pressure are
a. 0°C and 1 atm *b.* 273 K and 101.3 kPa
c. 298 K and 1 atm *d.* 0°C and 101.3 kPa

90. How many chiral carbon atoms are present in 2, 3, 4-trichloropentane ?
a. 3 *b.* 2 *c.* 1 *d.* 4

91. The number of unidentate ligands in the complex ion is called
a. EAN *b.* coordination number
c. primary valency *d.* oxidation number

92. $2SO_2(g) + O_2(g) \overset{V_2O_5}{\rightleftharpoons}$ is an example for
a. irreversible reaction
b. heterogeneous catalysis
c. homogeneous catalysis
d. neutralisation reaction

93. The amino acid which is not optically active is
a. glycine *b.* alanine
c. serine *d.* lactic acid

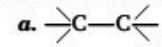
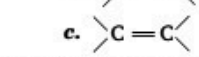
94. For a stable molecule the value of bond order must be
a. negative
b. positive
c. zero
d. there is no relationship between stability and bond order

95. Which one of the following is a second order reaction?
a. $CH_3COOCH_3 + NaOH \longrightarrow CH_3COONa + H_2O$
b. $H_2 + Cl_2 \xrightarrow{\text{Sunlight}} 2HCl$
c. $NH_4NO_3 \longrightarrow N_2 + 3H_2O$
d. $H_2 + Br_2 \longrightarrow 2HBr$

96. According to Baeyer's strain theory which is highly stable?
a. cyclohexane *b.* cycloheptane
c. cyclopentane *d.* cyclobutane

97. The number of antibonding electron pairs in O_2^{2-} molecular ion on the basis of molecular orbital theory is
[Atomic number of O is 18]
a. 2 *b.* 3 *c.* 4 *d.* 5

98. Hydroxyl ion concentration of 1M HCl is
a. 1×10^{-14} mol dm^{-3} *b.* 1×10^{-1} mol dm^{-3}
c. 1×10^{-13} mol dm^{-3} *d.* 1×10^{1} mol dm^{-3}

99. Geometrical isomerism is shown by
a.  *b.* —C≡C—
c.  *d.* None of these

100. The oxidation state of iron in $K_4[Fe(CN)_6]$ is,
a. 2 *b.* 3
c. 4 *d.* 1

101. In which of the following process, a maximum increase in entropy is observed?
a. dissolution of salt in water
b. condensation of water
c. sublimation of naphthalene
d. melting of ice

102. Decomposition of benzene diazonium chloride by using Cu_2Cl_2/HCl to form chlorobenzene is
a. Cannizarro's reaction
b. Kolbe's reaction
c. Sandmeyer's reaction
d. Raschig's reaction

103. Which complex cannot ionise in solution?
a. $[Pt(NH_3)_6]Cl_4$ *b.* $K_2[Pt(F_6)]$
c. $K_4[Fe(CN)_6]$ *d.* $[CoCl_3(NH_3)_3]$

104. Considering the reaction,
$C(s)+O_2(g)\longrightarrow CO_2(g)+393.5$ kJ
The signs of ΔH, ΔS and ΔG respectively are
a. −, +, − *b.* −, −, −
c. −, +, + *d.* +, −, −

105. The product formed when hydroxylamine condenses with a carbonyl compound is called
a. hydrazone *b.* hydrazine
c. oxime *d.* hydrazide

106. Which of the following forms a colourless solution in aqueous medium?
a. Ti^{3+} *b.* Sc^{3+}
c. V^{3+} *d.* Cr^{3+}

107. When a sulphur sol is evaporated, sulphur is obtained. On mixing with water, sulphur sol is not formed. The sol is
a. hydrophilic *b.* hydrophobic
c. reversible *d.* lyophilic

108. An alkyl halide reacts with alcoholic ammonia in a sealed tube, the product formed will be
a. a primary amine *b.* a secondary amine
c. a tertiary amine *d.* a mixture of all the three

109. When conc. H_2SO_4 is heated with P_2O_5, the acid is converted into
a. sulphur
b. sulphur dioxide
c. sulphur trioxide
d. a mixture of sulphur dioxide and sulphur trioxide

110. Entropy of the universe is
a. continuously increasing *b.* continuously decreasing
c. zero *d.* constant

111. Which of the following salts on being dissolved in water gives pH > 7 at 25°C ?
a. NH_4CN *b.* NH_4Cl *c.* KNO_3 *d.* KCN

112. The reagent used in Clemmenson's reduction is
a. alc. KOH *b.* *aq.* KOH
c. Zn-Hg/ conc. HCl *d.* conc. H_2SO_4

113. When KBr is dissolved in water, K^+ ions are
a. oxidised *b.* reduced
c. hydrolysed *d.* hydrated

114. The noble gas mixture is cooled in a coconut bulb at 173 K. The gases that are not adsorbed are
a. He and Ne *b.* Ar and Kr
c. He and Xe *d.* Ne and Xe

115. The volume of 10N and 4N HCl required to make 1 litre of 7N HCl are
a. 0.75 litre of 10N HCl and 0.25 litre of 4N HCl
b. 0.80 litre of 10 N HCl and 0.20 litre of 4N HCl
c. 0.60 litre of 10 N HCl and 0.40 litre of 4N HCl
d. 0.50 litre of 10 N HCl and 0.50 litre of 4N HCl

116. A metal present in insulin is
a. copper *b.* iron
c. zinc *d.* aluminium

117. Carbon forms two oxides which have different compositions. The equivalent mass of which remains constant?
a. Carbon
b. Oxygen
c. Neither carbon nor oxygen
d. Both carbon and oxygen

118. Maximum number of molecules of CH_3I that can react with a molecule of CH_3NH_2 are
a. 1 *b.* 2 *c.* 4 *d.* 3

119. Ellingham diagram represents a graph of
a. ΔG *vs* T *b.* $\Delta G°$ *vs* T
c. ΔS *vs* p *d.* ΔG *vs* T

120. Identify the ore not containing iron
a. chalcopyrites *b.* carnallite
c. siderite *d.* limonite

MATHEMATICS

121. $7^{2\log_7 5}$ is equal to

a. 5 *b.* $\log_7 35$

c. $\log_7 25$ *d.* 25

122. In the group $(G \otimes_{15})$, where $G = \{3, 6, 9, 12\}$, $\otimes_{15}$ is multiplication modulo 15, the identity element is

a. 6 *b.* 3

c. 9 *d.* 12

123. A group $(G *)$ has 10 elements. The minimum number of elements of G, which are their own inverses is

a. 1 *b.* 2

c. 0 *d.* 9

124. If $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is

a. 60° *b.* 120°

c. 30° *d.* 90°

125. $\dfrac{3x^2+1}{x^2-6x+8}$ is equal to

a. $\dfrac{49}{2(x-4)} - \dfrac{13}{2(x-2)}$ *b.* $3 + \dfrac{49}{2(x-4)} - \dfrac{13}{2(x-2)}$

c. $\dfrac{49}{2(x-4)} + \dfrac{13}{2(x-2)}$ *d.* $\dfrac{-49}{2(x-4)} + \dfrac{13}{2(x-2)}$

126. If $\mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$, $\mathbf{c} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - \hat{\mathbf{k}}$, then a vector perpendicular to $\mathbf{a}$ and in the plane containing $\mathbf{b}$ and $\mathbf{c}$ is

a. $17\hat{\mathbf{i}} + 21\hat{\mathbf{j}} - 123\hat{\mathbf{k}}$ *b.* $-17\hat{\mathbf{i}} + 21\hat{\mathbf{j}} - 97\hat{\mathbf{k}}$

c. $-17\hat{\mathbf{i}} - 21\hat{\mathbf{j}} - 97\hat{\mathbf{k}}$ *d.* $-17\hat{\mathbf{i}} - 21\hat{\mathbf{j}} + 97\hat{\mathbf{k}}$

127. $\mathbf{OA}$ and $\mathbf{BO}$ are two vectors of magnitudes 5 and 6 respectively. If $\angle BOA = 60°$, then $\mathbf{OA} \cdot \mathbf{OB}$ is equal to

a. 15 *b.* 0

c. $15\sqrt{3}$ *d.* -15

128. A vector perpendicular to the plane containing the points $A(1, -1, 2)$, $B(2, 0, -1)$, $C(0, 2, 1)$ is

a. $8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ *b.* $4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

c. $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ *d.* $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

129. $\dfrac{1}{2\cdot5} + \dfrac{1}{5\cdot8} + \dfrac{1}{8\cdot11} + \ldots \dfrac{1}{(3n-1)(3n+2)} =$

a. $\dfrac{n}{6n+3}$ *b.* $\dfrac{n}{6n-4}$

c. $\dfrac{n+1}{6n+4}$ *d.* $\dfrac{n}{6n+4}$

130. The ninth term of the expansion $\left(3x - \dfrac{1}{2x}\right)^8$ is

a. $\dfrac{-1}{512x^9}$ *b.* $\dfrac{1}{512x^9}$

c. $\dfrac{1}{256 \cdot x^8}$ *d.* $\dfrac{-1}{256 \cdot x^8}$

131. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ and B is the inverse of A, then the value of α is

a. 0 *b.* 2

c. 4 *d.* 5

132. If $A = \begin{bmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{bmatrix}$ is singular, then the possible values of x are

a. 0, 1, −1 *b.* 0, +12, −12

c. 0, 5, −5 *d.* 0, 4, −4

133. If $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, then $A \cdot \text{adj}(A)$ is equal to

a. $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ *b.* $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ *d.* $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

134. If $f : R \to R$ is defined by $f(x) = |x|$, then

a. $f^{-1}(x) = \dfrac{1}{|x|}$

b. $f^{-1}(x) = -x$

c. $f^{-1}(x) = \dfrac{1}{x}$

d. The function $f^{-1}(x)$ does not exist.

135. The value of $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$ is

a. $(x-p)(x-q)(x+p+q)$

b. $x(x-p)(x-q)$

c. $pq(x-p)(x-q)$

d. $(p-q)(x-q)(x-p)$

136. The number of common tangents to the circles $x^2+y^2=4$ and $x^2+y^2-6x-8y-24=0$ is,

a. 4 *b.* 3 *c.* 1 *d.* 2

137. If $3x+y+k=0$ is a tangent to the circle $x^2+y^2=10$, the values of k are

a. ± 5 *b.* ± 7 *c.* ± 9 *d.* ± 10

138. The negation of the proposition "If 2 is prime, then 3 is odd" is

a. 2 is prime and 3 is not odd.
b. If 2 is not prime then 3 is not odd.
c. If 2 is not prime then 3 is odd.
d. 2 is not prime and 3 is odd.

139. The equation to two circles which touch the Y-axis at (0, 3) and make an intercept of 8 units on X-axis are

a. $x^2+y^2\pm 6x-10y+9=0$
b. $x^2+y^2\pm 10x-6y+9=0$
c. $x^2+y^2+10x\pm 6y+9=0$
d. $x^2+y^2-8x\pm 10y+9=0$

140. The orthocentre of the triangle with vertices $A(0,0)$, $B(0,3/2)$, $C(-5,0)$ is

a. $(-5/2, 3/4)$ *b.* (5/2, 3/4)
c. (0, 0) *d.* $(-5, 3/2)$

141. $x^2+y^2-6x-6y+4=0$,
$x^2+y^2-2x-4y+3=0$,
$x^2+y^2+2kx+2y+1=0$. If the Radical centre of the above three circles exists, then which of the following cannot be the value of k?

a. 1 *b.* 2
c. 4 *d.* 5

142. If the circles $x^2+y^2-2x-2y-7=0$ and $x^2+y^2+4x+2y+k=0$ cut orthogonally, then the length of the common chord of the circles is

a. 2 *b.* $12/\sqrt{13}$
c. 8 *d.* 5

143. The coordinates of the foot of the perpendicular drawn from the point (3, 4) on the line $2x+y-7=0$ is

a. (1, 5) *b.* $\left(\frac{9}{5}, \frac{17}{5}\right)$
c. $(1, -5)$ *d.* $(-5, 1)$

144. The area enclosed by the pair of lines $xy=0$, the line $x-4=0$ and $y+5=0$ is

a. 10 sq units *b.* 20 sq units
c. 0 sq units *d.* $\frac{5}{4}$ sq units

145. If the area of the auxillary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 (a>b)$ is twice the area of the ellipse, then the eccentricity of the ellipse is

a. $\frac{\sqrt{3}}{2}$ *b.* $\frac{1}{\sqrt{2}}$ *c.* $\frac{1}{2}$ *d.* $\frac{1}{\sqrt{3}}$

146. A graph G has m vertices of odd degree and 'n' vertices of even degree. Then which of the following statements is necessarily true?

a. $m+n$ is an even number
b. $m+n$ is an odd number
c. $m+1$ is an odd number
d. $n+1$ is an even number

147. If p is any point on the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$, and S and S' are the foci, then $PS+PS'=$

a. 8 *b.* 4
c. 12 *d.* 10

148. The value of $\sin\left[2\cos^{-1}\frac{\sqrt{5}}{3}\right]$ is

a. $\frac{2\sqrt{5}}{3}$ *b.* $\frac{\sqrt{5}}{3}$ *c.* $\frac{2\sqrt{5}}{9}$ *d.* $\frac{4\sqrt{5}}{9}$

149. If $\frac{x^2}{36}-\frac{y^2}{k^2}=1$ is a hyperbola, then which of the following statements can be true?

a. (3, 1) lies on the hyperbola.
b. $(-3, 1)$ lies on the hyperbola.
c. (5, 2) lies on the hyperbola.
d. (10, 4) lies on the hyperbola.

150. The focus of the parabola is

a. $\left(\frac{1}{3}, \frac{-3}{2}\right)$ *b.* $\left(\frac{-1}{3}, \frac{3}{2}\right)$
c. $\left(\frac{1}{3}, \frac{-1}{2}\right)$ *d.* $\left(\frac{1}{3}, \frac{3}{2}\right)$

151. The solution of $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$ is

a. $\frac{1}{\sqrt{3}}$ *b.* $-\frac{1}{\sqrt{3}}$
c. $\sqrt{3}$ *d.* $-\sqrt{3}$

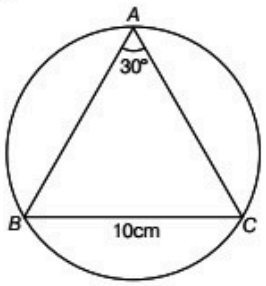
152. $\sin^2 175^\circ+\sin^2 725^\circ$ is equal to

a. $\tan^2 45^\circ$ *b.* $\cos^2 90^\circ$
c. $\sin^2 45^\circ$ *d.* $\cos^2 30^\circ$

153. The conjugate of the complex number $\frac{(1+i)^2}{1-i}$ is

a. $1+i$ *b.* $1-i$ *c.* $-1-i$ *d.* $-1+i$

154. ABC is a triangle with $\angle A = 30^\circ$ and $BC = 10$ cm. The area of the circum circle of the triangle is



a. 5 sq cm *b.* 100 π sq cm
c. $\frac{100\pi}{3}$ sq cm *d.* 25 sq cm

155. If $\sin 3\theta = \sin\theta$, how many solutions exist such that $-2\pi < \theta < 2\pi$?
a. 9 *b.* 8 *c.* 7 *d.* 5

156. The imaginary part of i^i is
a. 1 *b.* 0
c. -1 *d.* 2

157. The amplitude of $(1+i)^5$ is
a. $\frac{-3\pi}{4}$ *b.* $\frac{3\pi}{4}$
c. $\frac{5\pi}{4}$ *d.* $\frac{-5\pi}{4}$

158. ABC is a triangle, G is the centroid, D is the mid-point of BC. If $A = (2, 3)$ and $G = (7, 5)$, then the point D is
a. $\left(\frac{19}{2}, 6\right)$ *b.* $\left(\frac{9}{2}, 4\right)$
c. $\left(8, \frac{13}{2}\right)$ *d.* $\left(\frac{11}{2}, \frac{11}{2}\right)$

159. $\lim_{x\to 1} \frac{\tan(x^2-1)}{x-1}$ is equal to
a. $\frac{1}{2}$ *b.* 2
c. $\frac{-1}{2}$ *d.* -2

160. If $y = 2^{\log x}$, then $\frac{dy}{dx}$ is
a. $2^{\log x} \cdot \log 2$ *b.* $\frac{2^{\log x}}{\log 2}$
c. $\frac{2^{\log x}\log 2}{x}$ *d.* $\frac{2^{\log x}}{x}$

161. If $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$, then $\frac{dy}{dx}$ is
a. $\frac{y+1}{x-1}$ *b.* $\frac{y-1}{x+1}$
c. $\frac{x-1}{y+1}$ *d.* $\frac{x-1}{y-1}$

162. If $y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}$, then $\frac{d^2y}{dx^2}$ is
a. $9y$ *b.* $-3\sqrt{1-y^2}$
c. $3\sqrt{1-y^2}$ *d.* $-9y$

163. If the function $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
a. 0 *b.* 1
c. -1 *d.* 1/2

164. If $1, \omega, \omega^2$ are the cube roots of unity, then $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$ is equal to
a. 0 *b.* 1 *c.* ω *d.* ω^2

165. If $x^x = y^y$, then $\frac{dy}{dx}$ is
a. $-\frac{x}{y}$ *b.* $-\frac{y}{x}$
c. $\frac{1+\log x}{1+\log y}$ *d.* $1 + \log\left(\frac{x}{y}\right)$

166. The point on the curve $y^2 = x$, the tangent at which makes an angle 45° with X-axis is
a. (1/2, 1/4) *b.* (1/4, 1/2)
c. (1/2, 1/2) *d.* (1/2, $-1/2$)

167. The length of the subtangent to the curve $x^2y^2 = a^4$ at $(-a, a)$ is
a. $2a$ *b.* $a/2$ *c.* $a/3$ *d.* a

168. The number of positive divisors of 252 is
a. 5 *b.* 9 *c.* 10 *d.* 18

169. The remainder obtained when 5^{124} is divided by 124 is
a. 0 *b.* 5 *c.* 1 *d.* 2

170. Which of the following is not a group with respect to the given operation?
a. The set of odd integers under addition.
b. The set of even integers under addition.
c. {1, -1} under multiplication.
d. {0} under addition.

171. The range in which $y = -x^2 + 6x - 3$ is increasing, is

a. $x > 3$ ***b.*** $x < 3$

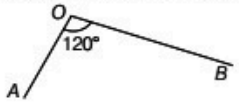
c. $5 < x < 6$ ***d.*** $7 < x < 8$

172. The value of the integral $\int_0^{\pi/2} (\sin^{100} x - \cos^{100} x)\, dx$ is

a. $\frac{100!}{(100)^{100}}$ ***b.*** $\frac{1}{100}$

c. 0 ***d.*** $\frac{\pi}{100}$

173. OA and OB are two roads enclosing an angle of $120°$. X and Y start from O at the same time. X travels along OA with a speed of 4 km/h and Y travels along OB with a speed of 3 km/h. The rate at which the shortest distance between X and Y is increasing after 1 hour is



a. 37 km/h ***b.*** $\sqrt{37}$ km/h

c. $\sqrt{13}$ km/h ***d.*** 13 km/h

174. If $k\int_0^1 x \cdot f(3x)\, dx = \int_0^3 t \cdot f(t)\, dt$, then the value of k is

a. 3 ***b.*** 9 ***c.*** 1/3 ***d.*** 1/9

175. The value of $\int \frac{1}{1 + \cos 8x}\, dx$ is

a. $\frac{\tan 8x}{8} + C$ ***b.*** $\frac{\tan 2x}{8} + C$

c. $\frac{\tan 4x}{8} + C$ ***d.*** $\frac{\tan 4x}{4} + C$

176. The value of $\int e^x (x^5 + 5x^4 + 1) \cdot dx$ is

a. $e^x \cdot x^5 + e^x + C$ ***b.*** $e^x \cdot x^5$

c. $5x^4 \cdot e^x$ ***d.*** $e^{x+1} \cdot x^5 + C$

177. The value of $\int \frac{x^2 + 1}{x^2 - 1}\, dx$ is

a. $\log\left(\frac{x+1}{x-1}\right) + C$ ***b.*** $\log\left(\frac{x-1}{x+1}\right) + C$

c. $\log(x^2 - 1) + C$ ***d.*** $x + \log\left(\frac{x-1}{x+1}\right) + C$

178. The area bounded by the curve $x = 4 - y^2$ and the Y-axis is

a. 32 sq units ***b.*** 16 sq units

c. $\frac{16}{3}$ sq units ***d.*** $\frac{32}{3}$ sq units

179. The differential equation of the family of straight lines whose slope is equal to y-intercept is

a. $(x+1)\frac{dy}{dx} + y = 0$

b. $(x+1)\frac{dy}{dx} - y = 0$

c. $\frac{dy}{dx} = \frac{x+1}{y+1}$

d. $\frac{dy}{dx} = \frac{x-1}{y-1}$

180. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^5\right]^{\frac{1}{3}} = \frac{d^2y}{dx^2}$ are respectively

a. 2, 1 ***b.*** 1, 5

c. 2, 3 ***d.*** 2, 5

ANSWERS

Physics

1.	(a)	2.	(c)	3.	(d)	4.	(b)	5.	(c)	6.	(a)	7.	(b)	8.	(d)	9.	(b)	10.	(b)
11.	(b)	12.	(c)	13.	(d)	14.	(a)	15.	(b)	16.	(c)	17.	(d)	18.	(b)	19.	(c)	20.	(a)
21.	(d)	22.	(d)	23.	(a)	24.	(a)	25.	(b)	26.	(a)	27.	(b)	28.	(c)	29.	(b)	30.	(c)
31.	(d)	32.	(d)	33.	(b)	34.	(d)	35.	(c)	36.	(b)	37.	(d)	38.	(a)	39.	(a)	40.	(b)
41.	(c)	42.	(c)	43.	(d)	44.	(c)	45.	(a)	46.	(b)	47.	(*)	48.	(c)	49.	(d)	50.	(a)
51.	(b)	52.	(d)	53.	(a)	54.	(b)	55.	(c)	56.	(c)	57.	(c)	58.	(c)	59.	(d)	60.	(a)

Chemistry

61.	(a)	62.	(b)	63.	(b)	64.	(d)	65.	(a)	66.	(b)	67.	(c)	68.	(d)	69.	(c)	70.	(c)
71.	(b)	72.	(b)	73.	(a)	74.	(c)	75.	(a)	76.	(c)	77.	(c)	78.	(a)	79.	(b)	80.	(c)
81.	(d)	82.	(a)	83.	(b)	84.	(d)	85.	(b)	86.	(d)	87.	(b)	88.	(c)	89.	(a)	90.	(b)
91.	(b)	92.	(b)	93.	(a)	94.	(b)	95.	(a)	96.	(c)	97.	(c)	98.	(a)	99.	(c)	100.	(a)
101.	(c)	102.	(c)	103.	(d)	104.	(a)	105.	(c)	106.	(b)	107.	(b)	108.	(d)	109.	(c)	110.	(a)
111.	(d)	112.	(c)	113.	(d)	114.	(a)	115.	(d)	116.	(c)	117.	(b)	118.	(d)	119.	(b)	120.	(b)

Mathematics

121.	(d)	122.	(a)	123.	(a)	124.	(d)	125.	(b)	126.	(c)	127.	(a)	128.	(a)	129.	(d)	230.	(c)
131.	(d)	132.	(b)	133.	(d)	134.	(d)	135.	(a)	136.	(c)	137.	(d)	138.	(a)	139.	(b)	140.	(a)
141.	(d)	142.	(b)	143.	(b)	144.	(b)	145.	(a)	146.	(a)	147.	(c)	148.	(d)	149.	(d)	150.	(*)
151.	(c)	152.	(a)	153.	(c)	154.	(b)	155.	(a)	156.	(b)	157.	(a)	158.	(a)	159.	(b)	160.	(c)
161.	(b)	162.	(d)	163.	(d)	164.	(b)	165.	(c)	166.	(b)	167.	(d)	168.	(d)	169.	(b)	170.	(a)
171.	(b)	172.	(c)	173.	(b)	174.	(b)	175.	(c)	176.	(a)	177.	(d)	178.	(d)	179.	(b)	180.	(c)

HINTS & SOLUTIONS

Physics

1. *(a)* Frequency of incident light,

$$\nu = 10^{15}\ \text{Hz}$$

Work function, $\phi_0 = 2.5$ eV

Energy of incident photon,

$$E = h\nu$$
$$= 6.62 \times 10^{-34} \times 10^{15}$$
$$= 6.62 \times 10^{-19}\ \text{J}$$
$$= \frac{6.62 \times 10^{-19}}{1.6 \times 10^{-19}}\ \text{eV}$$
$$= 4.1\ \text{eV}$$

According to Einstein's photoelectric equation,

$$K_{max} = h\nu - \phi_0$$
$$\Rightarrow \quad eV_0 = h\nu - \phi_0$$
$$\Rightarrow \quad eV_0 = 4.1eV - 2.5eV$$
$$\Rightarrow \quad eV_0 = 1.6eV$$
$$\Rightarrow \quad V_0 = 1.6\ \text{V}$$

2. *(c)* When a charged particle of charge q is accelerated through a potential V, then de-Broglie wavelength is given as

$$\lambda = \frac{h}{\sqrt{2mqV}}$$
$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{mq}}$$
$$\Rightarrow \quad \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p}{m_\alpha}} \cdot \sqrt{\frac{q_p}{q_\alpha}}$$
$$\Rightarrow \quad \frac{\lambda_\alpha}{\lambda} = \sqrt{\frac{m_p}{4m_p}} \sqrt{\frac{q_p}{2q_p}}$$

$\left[\because m_\alpha = 4m_p \text{ and } q_\alpha = 2q_p\right]$

$$\Rightarrow \quad \frac{\lambda_\alpha}{\lambda} = \frac{1}{\sqrt{8}}$$
$$\Rightarrow \quad \lambda_\alpha = \frac{\lambda}{2\sqrt{2}}$$

3. *(d)* Electric field due to thin spherical shell at a distance x outside from the centre is given as

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2} \qquad \ldots (i)$$

where, $Q = s \cdot A$

$$= s \times 4\pi R^2$$

$\therefore$ From Eq. (i), we get

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{s \times 4\pi R^2}{x^2} = \frac{sR^2}{\varepsilon_0 x^2}$$

$$\therefore \quad E \propto \frac{1}{x^2}$$

4. *(b)* For emission of energy, electron of H-atom must fall from higher energy state to lower energy state. Hence, options (a) and (c) are not possible.

In option (b), $n_1 = 2$, $n_2 = 1$

$\therefore$ Energy of emitted photon is given as

$$E' = -13.6\, Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$\Rightarrow \quad h\nu' = -13.6Z^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right)$$
$$\nu' = \frac{13.6Z^2}{h} \times \frac{3}{4} \qquad \ldots (i)$$

In option (c), $n_1 = 2$, $n_2 = 6$, hence energy of emitted photon is given as

$$E'' = -13.6\, Z^2\left(\frac{1}{6^2} - \frac{1}{2^2}\right)$$
$$\Rightarrow \quad h\nu'' = 13.6Z^2 \times \frac{8}{36}$$
$$\Rightarrow \quad \nu'' = \frac{13.6Z^2}{h} \times \frac{8}{36} \qquad \ldots (ii)$$

Hence, from Eqs. (i) and (ii), we get

$$\nu' > \nu''$$

5. *(c)* Nuclear force F_n are attractive in nature. As in the given case, the two protons are kept at a separation of 40 Å (short range), hence in this range, nuclear force F_n is much greater than the electrostatic force F_e.

i.e. $F_n >> F_e$

6. *(a)* Decays constants of radioactive materials X_1 and X_2 is given as $\lambda_1 = 5\lambda$ and $\lambda_2 = \lambda$

If N_0 be the initial number of nuclei in both radioactive materials, then according to radioactive decay law,

$$N_{X_1} = N_0 e^{-\lambda_1 t}$$

and $$N_{X_2} = N_0 e^{-\lambda_2 t}$$

$$\therefore \quad \frac{N_{X_1}}{N_{X_2}} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}}$$

$$\frac{1}{e} = e^{(-\lambda_1 + \lambda_2)t}$$

$$\left[\text{Given, } \frac{N_{X_1}}{N_{X_2}} = \frac{1}{e}\right]$$

$$\Rightarrow \quad e^{-1} = e^{-(\lambda_1 - \lambda_2)t}$$

$$\Rightarrow \quad (\lambda_1 - \lambda_2)t = 1$$

$$\Rightarrow \quad t = \frac{1}{\lambda_1 - \lambda_2}$$

$$= \frac{1}{5\lambda - \lambda} = \frac{1}{4\lambda}$$

7. *(b)* If α and β be the longitudinal strain and lateral strain respectively, then

$$\alpha = \frac{l_2 - l_1}{l_1} = 10^{-3} = 0.001$$

$$\Rightarrow \quad \frac{l_2}{l_1} = 1.001 \qquad \ldots \text{(i)}$$

$$\text{Poisson's ratio, } \sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\beta}{\alpha}$$

$$\Rightarrow \quad \beta = \sigma\alpha = 0.1 \times 10^{-3} = 10^{-4}$$

$$\text{But,} \quad \beta = \frac{r_1 - r_2}{r_1}$$

$$\therefore \quad \frac{r_1 - r_2}{r_1} = 10^{-4} = 0.0001$$

$$\Rightarrow \quad \frac{r_2}{r_1} = 0.9999 \qquad \ldots \text{(ii)}$$

Hence, percentage increase in volume

$$= \frac{V_2 - V_1}{V_1} \times 100$$

$$= \frac{\pi r_2^2 l_2 - \pi r_1^2 l_1}{\pi r_1^2 l_1} \times 100$$

$$= \left[\left(\frac{r_2}{r_1}\right)^2 \left(\frac{l_2}{l_1}\right)^2 - 1\right] \times 100$$

$$= [(0.9999)^2 \times (1.0001)^2 - 1] \times 100$$

$$= 0.0008 \times 100 = 0.08\%$$

8. *(d)* We know that, angular velocity,

$$\omega = \frac{2\pi}{T}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2}$$

$$= \frac{\omega_1}{2\omega_1} \qquad [\because \omega_2 = 2\omega_1]$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{1}{2}$$

$$\Rightarrow \quad T_2 = \frac{T_1}{2} \qquad \ldots \text{(i)}$$

According to Kepler's law,

$$T^2 \propto R^3$$

$$\Rightarrow \quad R \propto T^{2/3}$$

$$\Rightarrow \quad \frac{R_2}{R_1} = \left(\frac{T_2}{T_1}\right)^{2/3}$$

$$= \left(\frac{T_1/2}{T}\right)^{2/3}$$

$$= \left(\frac{1}{2}\right)^{2/3}$$

$$R_2 = \frac{R_1}{2^{2/3}}$$

$$= \frac{R}{4^{1/3}}$$

9. *(b)* When a *p-n* junction diode is connected in forward bias, then its potential barrier decreases because more number of charge carriers (electrons in *n*-side and holes in *p*-side) move through the junction. Due to increase of drifting of charge carriers, electric current is increased.

10. *(b)* Wavelength of ground waves have frequency greater than the wavelength of sky waves and space waves both.

11. *(b)* In the given situation, the weight of the ball is supported by the buoyant force of water and buoyant force due to air. As the density of air becomes less, the buoyant force due to the air becomes less. Therefore, to compensate a greater buoyant force due to water is required which is achieved by a greater immersion of ball into the water.

12. *(c)* Given, initial surface area, $A_1 = A$

Final surface area, $A_2 = A_1 - 50\%$ of A

$$A_2 = A - 50\% \text{ of } A = A - \frac{A}{2} = \frac{A}{2}$$

Initial surface energy, $E_1 = T \cdot 2A = 2TA$ … (i)

Final surface energy, E_2 = Surface tension × Surface area

$$\Rightarrow \quad E_2 = T \times 2(A_1 - A_2)$$

$$= T \times 2\left(A - \frac{A}{2}\right)$$

$$= T \times 2 \times \frac{A}{2}$$

$\Rightarrow \quad E_2 = TA$

$\therefore$ Percentage change in the energy of soap film

$$= \frac{E_1 - E_2}{E_1} \times 100$$

$$= \frac{2TA - TA}{2TA} \times 100$$

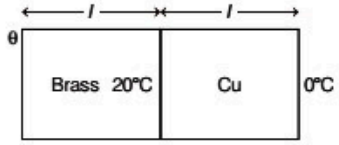
$$= 50\%$$

13. *(d)* Given, $K_{copper} : K_{brass} = 4 : 1$

$$\Rightarrow \quad \frac{K_{copper}}{K_{brass}} = \frac{4}{1}$$

Let the temperature of the free surface of brass = θ

Rate of flow of heat will be equal in both the plates as connected in series.



i.e. $H = K_{brass} \frac{A(\theta - 20)}{l} = K_{copper} \frac{A(20 - 0)}{l}$

$$\Rightarrow \quad \frac{\theta - 20}{20} = \frac{K_{copper}}{K_{brass}} = \frac{4}{1}$$

$$\Rightarrow \quad \theta - 20 = 80$$

$$\theta = 100°C$$

14. *(a)* Given, $V_1 = 1 \text{ mm}^3$

$$= 10^{-9} \text{ m}^3$$

$$p_1 = 1 \text{ atm}$$

$$= 1 \times 10^5 \text{ N/m}^2$$

$$T_1 = 27 + 273$$

$$= 300 \text{ K}$$

$$T_2 = 627 + 273$$

$$= 900 \text{ K}$$

Under adiabatic condition,

$$p_1^{1-\gamma} T_1^{\gamma} = p_2^{1-\gamma} T_2^{\gamma}$$

$$\Rightarrow \quad \left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma}$$

$$\Rightarrow \quad \left(\frac{p_2}{p_1}\right)^{1-1.5} = \left(\frac{300}{900}\right)^{1.5}$$

$$\Rightarrow \quad \left(\frac{p_2}{p_1}\right)^{-0.5} = \left(\frac{1}{3}\right)^{1.5}$$

$$\Rightarrow \quad \left(\frac{p_1}{p_2}\right)^{0.5} = \left(\frac{1}{3}\right)^{1.5}$$

$$\Rightarrow \quad \frac{p_1}{p_2} = \left(\frac{1}{3}\right)^{\frac{1.5}{0.5}} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$\Rightarrow \quad p_2 = 27\, p_1 = 27 \times 1 \times 10^5$$

$$= 27 \times 10^5 \text{ N/m}^2$$

15. *(b)* Given, temperature of sink,

$$T_2 = -39°C$$

$$= (273 - 39) \text{ K}$$

$$= 234 \text{ K}$$

Temperature of source,

$$T_1 = 0°C$$

$$= (273 + 0) \text{ K}$$

$$= 273 \text{ K}$$

$\therefore$ Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

$$= 1 - \frac{234}{273}$$

$$= \frac{39}{273}$$

$$= 0.142$$

$$= 14.2\%$$

16. *(c)* Law of conservation of energy is valid across all domains of nature. In the universe, all kinds of violent phenomenon occur all the time. Yet the total energy of the universe is remains conserved.

17. *(d)* Given, centripetal force, $F = -\frac{A}{R^2}$

$\therefore$ Potential energy, $U = -\frac{dF}{dR}$

$$= -\frac{d}{dR}\left(-\frac{A}{R^2}\right)$$

$$= -\frac{A}{R}$$

In circular motion, centripetal force is responsible for the motion of object.

$\therefore \quad \frac{mv^2}{R} = \frac{A}{R^2}$

$\Rightarrow \quad mv^2 = \frac{A}{R}$

$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{A}{2R}$

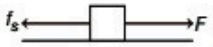
$\therefore$ Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{A}{2R}$

$\therefore$ Total energy = $U + K$

$$= -\frac{A}{R} + \frac{A}{2R} = -\frac{A}{2R}$$

18. *(b)* Given, force $F = 20$ N

$m = 5$ kg



Net force on the body,

$F_{net} = F - f_s$ [where, f_s = friction]

$\therefore$ Work done by displacing 2 m is given as

$W = F_{net} \times$ displacement

$= (F - f_s)2$

$= (20 - f_s)2$

$\Rightarrow \quad W = 40 - 2f_s$

According to work-energy theorem,

Work done = gain in kinetic energy

$\Rightarrow \quad 40 - 2f_s = 10$

$\Rightarrow \quad 2f_s = 40 - 10 = 30$

$\Rightarrow \quad f_s = \frac{30}{2}$

$= 15$ N

19. *(c)* Given, $\mathbf{F} = 6\hat{\mathbf{i}} - 8\hat{\mathbf{j}} + 10\hat{\mathbf{k}}$

$a = 1$ m/s^2

$|\mathbf{F}| = \sqrt{6^2 + (-8)^2 + 10^2}$

$= \sqrt{200}$

$\Rightarrow \quad |\mathbf{F}| = 10\sqrt{2}$ N

$\Rightarrow \quad ma = 10\sqrt{2}$ $[\because F = ma]$

$\Rightarrow \quad m = \frac{10\sqrt{2}}{a}$

$= \frac{10\sqrt{2}}{1}$

$= 10\sqrt{2}$ kg

20. *(a)* Mass of the body, $m = 1000$ kg

$v = 50$ m/s

After adding 250 kg, new mass of the body,

$m' = m + 250$

$= 1000 + 250$

$= 1250$ kg

If v' be the final velocity of the body, then by conservation of linear momentum,

$mv = m'v'$

$\Rightarrow 1000 \times 50 = 1250 \times v'$

$\Rightarrow \quad v' = \frac{1000 \times 50}{1250}$

$= 40$ m/s

21. *(d)* Densities ratio of two gases,

$\rho_1 : \rho_2 = 1 : 16$

$\Rightarrow \quad \frac{\rho_1}{\rho_2} = \frac{1}{16}$

Since, pressure of both gases are equal.

$\therefore \quad p_1 = p_2$

$\frac{1}{3}\rho_1 c_1^2 = \frac{1}{2}\rho_2 c_2^2$

$\Rightarrow \quad \frac{c_1^2}{c_2^2} = \frac{\rho_2}{\rho_1} = \frac{16}{1}$

$= \left(\frac{4}{1}\right)^2$

$\Rightarrow \quad \frac{c_1}{c_2} = \frac{4}{1}$

$\Rightarrow \quad c_1 : c_2 = 4 : 1$

22. *(d)* Mass of helium, $m_1 = 16$ g

Mass of oxygen, $m_2 = 16$ g

$\therefore$ Number of moles of He gas,

$n_1 = \frac{m_1}{4} = \frac{16}{4} = 4$

Number of moles of O_2 gas,

$n_2 = \frac{m_2}{32} = \frac{16}{32} = \frac{1}{2}$

We know that for the mixture,

$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$, where $C_V = \frac{fR}{2}$

and $C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$, where $C_p = \left(\frac{f}{2} + 1\right)R$

For He gas, $f = 3$, $n_1 = 4$

For O_2 gas, $f = 5$, $n_2 = \frac{1}{2}$

$\therefore \quad \frac{C_p}{C_V} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$

$= \frac{4 \times \left(\frac{3}{2} + 1\right)R + \frac{1}{2}\left(\frac{5}{2} + 1\right)R}{4 \times \frac{3}{2}R + \frac{1}{2} \times \frac{5}{2}R}$

$= \frac{\left(4 \times \frac{5}{2}R\right) + \left(\frac{1}{2} \times \frac{7}{2}R\right)}{\left(4 \times \frac{3}{2}R\right) + \left(\frac{1}{2} \times \frac{5}{2}R\right)} = \frac{47}{29}$

$= 1.62$

23. *(a)* Amplitude of the particle is SHM,

$A = 4$ cm

At mean position velocity,

$v_{max} = 10$ cm/s

$\Rightarrow \quad A\omega = 10$

$$\Rightarrow \quad \omega = \frac{10}{A}$$

$$= \frac{10}{4} = 2.5 \text{ rad/s}$$

Again when $v = 5$ cm/s, then displacement, $x = ?$

We know that,

$$v = \omega\sqrt{A^2 - x^2}$$

$$\Rightarrow \quad 5 = 25\sqrt{4^2 - x^2}$$

$$\Rightarrow \quad 2 = \sqrt{A^2 - x^2}$$

$$\Rightarrow \quad 4 = A^2 - x^2$$

$$\Rightarrow \quad 4 = 4^2 - x^2$$

$$\Rightarrow \quad x^2 = 16 - 4 = 12$$

$$\therefore \quad x = \sqrt{12} = 2\sqrt{3} \text{ cm}$$

24. *(a)* Given, equation of progressive wave,

$$y = 15\sin(660\pi t - 0.02\,\pi x)$$

Comparing this equation with standard equation of progressive wave as $y = A\sin\left(\frac{2\pi}{T}.t - \frac{2\pi}{\lambda}.x\right)$

We get,

$$\frac{2\pi}{T} = 660\pi$$

$$\Rightarrow \quad T = \frac{2\pi}{660\pi}$$

$$\Rightarrow \quad T = \frac{1}{330}$$

$\therefore$ Frequency of the wave,

$$\nu = \frac{1}{T}$$

$$= \frac{1}{1/330} = 330 \text{ Hz}$$

25. *(b)* Let the frequency of the sound is f.

In first case, $f = 100 \pm 5$

$= 105$ Hz or 95 Hz

The frequency of second harmonic $= 2f$

According to second case,

$$2f = 200 \pm 10$$

$= 210$ Hz or 190 Hz

If 5 beats are produced with frequency 105 Hz, then frequency of second harmonic will be 210 Hz.

i.e. $\quad 2f = 210$

$$\Rightarrow \quad f = 105 \text{ Hz}$$

$$= 105 \text{ s}^{-1}$$

26. *(a)* Given, $Q_1 + Q_2 = 0.8\,\text{C}$

$$r = 30 \text{ cm} = 0.3 \text{ m}$$

$\therefore$ According to Coulomb's law

$$F = k\frac{Q_1 Q_2}{r^2}$$

$$\Rightarrow \quad F = k\frac{Q_1(0.8 - Q_1)}{r^2}$$

The force on Q_1 will be maximum if

$$\frac{dF}{dQ_1} = 0$$

$$\Rightarrow \quad \frac{d}{dQ}k\frac{Q_1(0.8 - Q_1)}{r^2} = 0$$

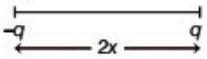
$$0.8 - 2Q_1 = 0$$

$$\Rightarrow \quad Q_1 = 0.4\,\text{C}$$

$$\therefore \quad Q_2 = 0.8 - Q_1$$

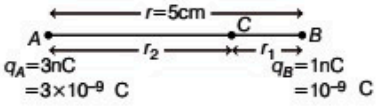
$$= 0.8 - 0.4 = 0.4\,\text{C}$$

27. *(b)* The axis of the electric dipole is directed along the direction of dipole moment and the direction of electric dipole moment is taken from negative charge to positive charge.



28. *(c)* The orientation of the dipole for the stable equilibrium, $\theta = 0°$.

29. *(b)* The given situation is shown below



$$r_1 = 1 \text{ cm}$$

$$= 1 \times 10^{-2} \text{ m}$$

$$r_2 = 5 - 1$$

$$= 4 \text{ cm}$$

$$= 4 \times 10^{-2} \text{ m}$$

Required work done = Changes in potential energy

$$= k q_A q_B\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= 9 \times 10^9 \times 3 \times 10^{-9} \times 10^{-9}$$

$$\left[\frac{1}{4 \times 10^{-2}} - \frac{1}{1 \times 10^{-2}}\right]$$

$$= 1.35 \times 10^{-7} \text{ J}$$

30. *(c)* We know that, potential energy of electric dipole,

$$U = -\mathbf{p}\cdot\mathbf{E} = -pE\cos\theta$$

When, $90 < \theta < 180$, then $\cos\theta$ is negative.

As θ increases, $\cos\theta$ decreases.

$\therefore \quad \theta_{(iv)} > \theta_{(iii)}$

When $0° < \theta \leq 90°$, then $\cos\theta$ is positive.

Also, when θ increases, then $\cos\theta$ decreases.

$\therefore$ Hence, $\theta_{(i)} > \theta_{(ii)}$

Therefore, $\theta_{(iv)} > \theta_{(iii)} > \theta_{(i)} > \theta_{(ii)}$.

31. *(d)* Refractive index, $\mu = A + \frac{B}{\lambda^2}$

According to principle of homogeneity,

Dimension of $\frac{B}{\lambda^2}$ = Dimension of μ

$$\Rightarrow \quad [B] = [\mu][\lambda^2]$$
$$= [M^0L^0T^0][L^2]$$
$$= [L^2]$$
$$= \text{Dimension of area}$$

32. *(d)* Voltage, $V = (200 \pm 8)$ V

Current, $I = (20 \pm 0.5)$ A

$\therefore$ Resistance, $R = \frac{V}{I} = \frac{200}{20} = 10\ \Omega$

Taking maximum error condition,

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{8}{200} + \frac{0.5}{20} = \frac{13}{200}$$

$$\Rightarrow \quad \frac{\Delta R}{R} \times 100 = \frac{13}{200} \times 100$$
$$= \frac{13}{2}\% = 6.5\%$$

33. *(b)* Let u be the initial velocity of vertical projection and t be the time taken by the body to reach at height h from the ground.

$\therefore$ Using equation, $h = ut + \frac{1}{2}at^2$

Here, $u = u$, $a = -g$

$$\therefore \quad h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow \quad gt^2 - 2ut + 2h = 0$$

$$\therefore \quad t = \frac{2u \pm \sqrt{4u^2 - 4g \times 2h}}{2g}$$
$$= \frac{u \pm \sqrt{u^2 - 2gh}}{g}$$

It means t has two values, t_1 and t_2. Where, t_1 is time taken by the body in reaching maximum height and t_2 be the time taken by the body in reaching ground from maximum height.

$$\therefore \quad t_1 = \frac{u + \sqrt{u^2 - 2gh}}{g}$$

and $$t_2 = \frac{u - \sqrt{u^2 - 2gh}}{g}$$

$$\therefore \quad t_1 + t_2 = \frac{2u}{g} \quad \text{or} \quad u = \frac{g(t_1 + t_2)}{2}$$

34. *(d)* Slope of distance-time graph gives the velocity of the particle. Since at the point C, slope of the curve is maximum, hence maximum instantaneous velocity of particle occurs at point C.

35. *(c)* According to the equation of continuity,

$$Av = \text{constant}$$

Therefore, velocity of the fluid is minimum at the widest part and maximum at the narrowest part of the horizontal pipe.

Again, according to Bernoulli's principle for horizontal pipe,

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

Hence, p is more, v is less and *vice-versa*.

Therefore, when a fluid is flowing through the horizontal pipe of variable area of cross-section, then velocity is maximum at the narrowest part of the pipe and pressure is maximum at the widest part of the pipe.

36. *(b)* Given, $V_1 = 120$ V, $V_2 = 200$ V

Since, potential on each capacitor is zero when they are connected in parallel. Hence, charge on each capacitor must be equal with unlike polarities together.

i.e. $$q_1 = q_2$$

$$\Rightarrow \quad C_1V_1 = C_2V_2$$

$$\Rightarrow \quad C_1 \times 120 = C_2 \times 200$$

$$\Rightarrow \quad 3C_1 = 5C_2$$

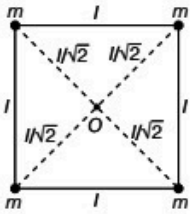
37. *(d)* According to Kirchhoff's junction rule,

$$\Sigma I = 0$$

i.e. sum of total incoming currents = sum of total outgoing currents.

Hence, it is based on the law of conservation of charges. Hence, all the statements are correct.

38. *(a)* The given situation is shown in figure.



Moment of inertia of the systems about an axis perpendicular to the square and passing through the centre is given as

$$I = m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= 4\frac{ml^2}{2} = 2ml^2 \qquad \ldots \text{(i)}$$

If k be the radius of gyration, then

$$I = 4mk^2 \qquad \ldots \text{(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we have

$$2ml^2 = 4mk^2$$

$$\Rightarrow \quad k = \frac{l}{\sqrt{2}}$$

39. *(a)* Given, for steel wire,

Area, $A_1 = 3 \times 10^{-5}\ \text{m}^2$

Length, $L_1 = 4.7\ \text{m}$

For copper wire,

$$A_2 = 4 \times 10^{-5}\ \text{m}^2,\ L_2 = 3.5\ \text{m}$$

If Y_1 and Y_2 be the Young's modulus for the material steel wire and copper wire respectively, then

$$\frac{Y_1}{Y_2} = \frac{\frac{F_1 L_1}{A_1 \Delta l}}{\frac{F_2 L_2}{A_2 \Delta l}}$$

$$\Rightarrow \quad \frac{Y_1}{Y_2} = \frac{L_1 A_2}{L_2 A_1} \qquad [\because F_1 = F_2]$$

$$= \frac{4.7 \times 4 \times 10^{-5}}{3.5 \times 3 \times 10^{-5}} = 1.8$$

$$\Rightarrow \quad Y_1 : Y_2 = 1.8 : 1$$

40. *(b)* Given, $m_1 = 200\ \text{g} = 0.2\ \text{kg}$

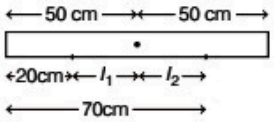
$m_2 = 300\ \text{g} = 0.3\ \text{kg}$

$l_1 = 50 - 20 = 30\ \text{cm}$

$$\Rightarrow \quad l_1 = 0.3\ \text{m}$$

$l_2 = 70 - 50 = 20\ \text{cm} = 0.2\ \text{m}$

$\therefore$ Moment of inertia of the system about an axis passing through 50 cm mark (i.e. mid-point)

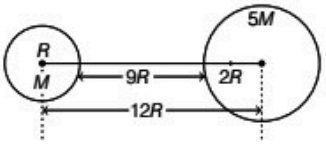


$$I = m_1 l_1^2 + m_2 l_2^2$$

$$= 0.2 \times (0.3)^2 + 0.3 \times (0.2)^2$$

$$= 0.036\ \text{kg m}^2$$

41. *(c)* The given situation is shown below.



Let the smaller body travels a distance x before collision due to gravitation attraction.

Hence, $Mx = 5M(9R - x)$

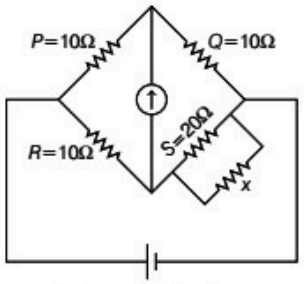
$$\Rightarrow \quad x = 5(9R - x)$$

$$\Rightarrow \quad x = 45R - 5x$$

$$\Rightarrow \quad 6x = 45R$$

$$\Rightarrow \quad x = \frac{45}{6}R = \frac{15}{2}R = 7.5R$$

42. *(c)* The given Wheatstone bridge circuit is shown below.



Wheatstone is balanced only when

$$\frac{P}{Q} = \frac{R}{S}$$

Thus, the value of resistance S should be 10 Ω. But it is given to 20 Ω.

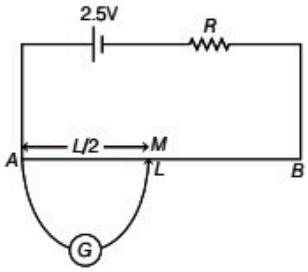
Hence, to decrease its value upto 10 Ω, suppose x Ω resistance is connected parallel to it.

$$\therefore \quad \frac{1}{10} = \frac{1}{20} + \frac{1}{x}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$$

$$\Rightarrow \quad x = 20\ \Omega$$

43. *(d)* The given situation is shown below.



Resistance of potentiometer wire, $R_P = 10\ \Omega$

Emf, $E = 25\ \text{V}$

Total resistance, $R' = R_P + R = (10 + R)\ \Omega$

Current through potentiometer wire,

$$I = \frac{E}{R_P} = \frac{25}{10 + R}$$

Since, potential difference across length $AL = \frac{L}{2}$,

$$V = I\left(\frac{R_P}{2}\right)$$

$$\Rightarrow \quad 1 = \frac{25}{10+R} \times \frac{10}{2}$$

$$\Rightarrow \quad 10 + R = 125$$

$$\Rightarrow \quad R = 25\ \Omega$$

44. *(c)* Given,

$$\mathbf{v} = 2 \times 10^5 \hat{\mathbf{i}}\ \text{m/s}$$

$$\mathbf{B} = (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}})\ \text{T}$$

Charge on electron, $q = 1.6 \times 10^{-19}$ C

$\therefore$ Force, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$

$$= 1.6 \times 10^{-19}[(2 \times 10^5 \hat{\mathbf{i}}) \times (\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 3\hat{\mathbf{k}})]$$

$$= 1.6 \times 10^{-19} \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 \times 10^5 & 0 & 0 \\ 1 & -4 & -3 \end{vmatrix}$$

$$\mathbf{F} = 1.6 \times 10^{-19}(6 \times 10^5\ \hat{\mathbf{j}} - 8 \times 10^5 \hat{\mathbf{k}})$$

$$= 9.6 \times 10^{-14}\ \hat{\mathbf{j}} - 12.8 \times 10^{-14}\ \hat{\mathbf{k}}$$

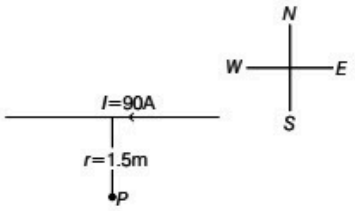
$$\Rightarrow \quad |\mathbf{F}| = \sqrt{(9.6 \times 10^{-14})^2 + (-12.8 \times 10^{-14})^2}$$

$$= 10^{-14}\sqrt{(9.6)^2 + (12.8)^2}$$

$$= 16 \times 10^{-14}\ \text{N}$$

$$= 1.6 \times 10^{-13}\ \text{N}$$

45. *(a)* The given situation is shown below



Magnetic field at point P,

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

$$= 2 \times 10^{-7} \times \frac{90}{1.5}$$

$$= 1.2 \times 10^{-5}\ \text{T (South)}$$

46. *(b)* Given, number of turns in moving coil galvanometer, $N_i = 28$

Initial sensitivity, $S_i = S$

Final sensitivity, $S_f = S_i + 25\%\ \text{of}\ S_i$

$$= S + 25\%\ \text{of}\ S$$

$$= S + \frac{S}{4} = \frac{5S}{4}$$

Since, sensitivity of moving coil galvanometer is given as

$$S = \frac{NAB}{K}$$

$$\Rightarrow \quad S \propto N \qquad [A, B \text{ and } K \text{ are constant}]$$

$$\Rightarrow \quad \frac{S_f}{S_i} = \frac{N_f}{N_i}$$

$$\Rightarrow \quad \frac{\frac{5S}{4}}{S} = \frac{N_f}{28}$$

$$\Rightarrow \quad N_f = \frac{28 \times 5}{4} = 35$$

47. *(*)* The intensity of magnetic field due to isolated pole of strength m_p at a point distance r from it is given as

$$B = \frac{\mu_0}{4\pi} \cdot \frac{m_p}{r^2}$$

Hence, no option is correct.

48. *(c)* Light particle (like electron) cannot be accelerated by a cyclotron because due its small mass, a small increase in its energy makes the electrons move with a very high speed. As a result of it, the electrons go quickly out of step with oscillation electric field.

49. *(d)* The angle which the total magnetic field of earth makes with the surface of the earth is called inclination or angle of dip.

50. *(a)* Given, $B_H = B_V$

If δ be the angle of dip, then

$$\tan\delta = \frac{B_V}{B_H} = \frac{B_V}{B_V} = 1$$

$$= \tan 45°$$

$$\Rightarrow \quad \delta = 45°$$

51. *(b)* Given, number of turns in the first coil, $N_1 = 100$

Its self-inductance, $L_1 = 15\ \text{mH} = 1.5 \times 10^{-2}$ H

Number of turns in the second coil,

$$N_2 = 500$$

$$\therefore \quad L_2 = ?$$

We know that, self inductance of the coil is given by

$$L = \frac{\mu_0 \pi N^2 R}{2}$$

$$\Rightarrow \quad L \propto N^2$$

$$\therefore \quad \frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2 = \left(\frac{500}{100}\right)^2 = 25$$

$$\Rightarrow \quad L_2 = 25\, L_1$$

$$= 25 \times 1.5 \times 10^{-2}$$

$$= 375 \times 10^{-2}\ \text{H}$$

$$= 375 \times 10^{-3}\ \text{H} = 375\ \text{mH}$$

52. *(d)* Given, number of turns, $N = 100$

Current, $I = 5\text{ mA} = 5 \times 10^{-3}\text{ A}$

Magnetic flux, $\phi = 10^{-5}\text{ Wb}$

Inductance, $L = ?$

We know that,

$$N\phi = LI$$

$$\Rightarrow \quad L = \frac{N\phi}{I} = \frac{100 \times 10^{-5}}{5 \times 10^{-3}}$$

$$= 20 \times 10^{-2}\text{ H} = 0.2\text{ H}$$

53. *(a)* In step-up transformer, number of turns in secondary coil is greater than the number of turns in the primary coil.

i.e. $N_S > N_P$

54. *(b)* In L-C-R series resonance circuit,

$$X_L = X_C$$

$$\therefore \text{ Impedance, } Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 0^2}$$

$$\Rightarrow \quad Z = R$$

$$\therefore \text{ Power factor, } \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

Which is not zero.

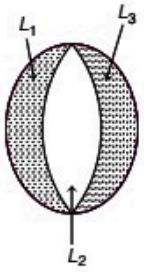
Since, circuit behaves like a resistive circuit. So, power loss occurs always in this circuits.

i.e. wattless current is zero.

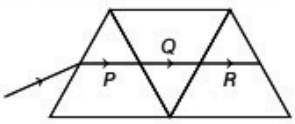
55. *(c)* Both magnet and electric fields have zero average value in a plane electromagnetic wave.

56. *(c)* From the given diagram, we see that lenses L_1 and L_3 are identical but contribute opposite sense, hence their combined effect of making image is ineffective.

Hence, only lens L_2 forms the image.



57. *(c)* The complete ray diagram for the given combination of prisms is shown below.



Since, ray of light incident on prism P suffers minimum deviation, hence it becomes parallel to the base of prism P. As prism Q and R are of identical shape and of same material as that of prism P, therefore, the ray continues to be parallel to base of prisms Q and R. Hence, the ray will now suffer same deviation as before.

58. *(c)* Phase difference between the two identical waves = δ

For identical wave,

$$a_1 = a_2 = a$$

After superposition, intensity of resultant wave

$$I \propto (\text{Amplitude})^2$$

$$\propto A^2$$

$$\propto (a_1^2 + a_2^2 + 2a_1a_2\cos\delta)$$

$$\propto (a^2 + a^2 + 2a^2\cos\delta)$$

$$\propto 2a^2(1 + \cos\delta)$$

$$\propto 2a^2 \cdot \left(2\cos^2\frac{\delta}{2}\right)$$

$$\propto 4a^2\cos^2\left(\frac{\delta}{2}\right)$$

$$\Rightarrow \quad I \propto \cos^2\left(\frac{\delta}{2}\right)$$

59. *(d)* Given, wavelength of monochromatic light, $\lambda = 5867\text{ Å}$

Refractive index, $\mu = 1.6$

Let t be the thickness of the plastic.

Since, the centre of the screen appears dark when plastic sheet is introduced, hence path difference,

$$\mu t = \frac{\lambda}{2} \text{ or } \frac{3\lambda}{2}$$

when, $\mu t = \dfrac{3\lambda}{2}$

$$\Rightarrow \quad t = \frac{3\lambda}{2\mu} = \frac{3 \times 5867}{2 \times 1.6} = 5500\text{ Å}$$

60. *(a)* Position vector of starting point (2, 10, 1) of particle is given as

$$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

If final co-ordinate of the particle be (x, y, z), then its position vector

$$\mathbf{r}_2 = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

$$\therefore \text{ Displacement, } \Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$

$$8\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} - (2\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$\Rightarrow \quad 8\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}} = (x - 2)\hat{\mathbf{i}} + (y - 10)\hat{\mathbf{j}} + (z - 1)\hat{\mathbf{k}}$$

Comparing the coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, we get

$$x - 2 = 8,\ y - 10 = -2,\ z - 1 = 1$$

$$\Rightarrow \quad x = 10,\ y = 8,\ z = 2$$

$\therefore$ Final co-ordinate = (10, 8, 2)

Chemistry

61. *(a)* During the extraction of gold, following reactions take place

$$\underset{\text{Impure gold}}{4Au(s)} + 8CN^-(aq) + 2H_2O(aq) \xrightarrow{O_2} \underset{(X)}{4[Au(CN)_2]^-} + 4OH^-(aq)$$

$$\underset{(X)}{2[Au(CN)_2]^-} + Zn(s) \longrightarrow \underset{(Y)}{[Zn(CN)_4]^{2-}}(aq) + \underset{\text{Pure gold}}{2Au(s)}$$

One chlorine molecule contains 2 chlorine atoms and one HCl molecule contains 1 chlorine atom.

62. *(b)* Number of gram molecules of chlorine in HCl

$$= \frac{\text{number of molecules of HCl}}{N_A \times 2}$$

$$= \frac{6.02 \times 10^{25}}{6.02 \times 10^{23} \times 2}$$

$$= \frac{100}{2} = 50$$

63. *(b)* Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds.

64. *(d)* Paracetamol is an analgesic and antipyretic drug. It is used in medication for treatment of fever, mild to moderate pain.

65. *(a)* 2g of water has maximum number of molecules as its molecular mass is minimum among the given compounds.

Number of O-atoms in $H_2O = \frac{2}{18}N_A = \frac{1}{9}N_A$

Number of O-atoms in $SO_2 = \frac{2}{64}N_A = \frac{1}{32}N_A$

Number of O-atoms in $CO_2 = \frac{2}{44} \times 2N_A = \frac{1}{11}N_A$

Number of O-atoms in $CO = \frac{2}{28}N_A = \frac{1}{14}N_A$

66. *(b)* C_2H_5OH (alcohol) shows functional group isomerism.

It's isomers are

$\underset{\text{Alcohol}}{C_2H_5OH}$, $\underset{\text{Ether}}{CH_3—O—CH_3}$

67. *(c)* For the following reaction,

$$BiO_3^- + 6H^+ + xe^- \longrightarrow Bi^{3+} + 3H_2O$$

On balancing charge on both sides

$$-1 + 6 + x(-1) = 3 + 3 \times 0$$

$$\therefore \quad x = 2$$

68. *(d)* For orthophosphoric acid solution,

Molarity = 3 M, *n*-factor = 3

$$\therefore \quad \text{Normality} = n\text{-factor} \times \text{molarity}$$

$$= 3 \times 3$$

$$= 9\ \text{N}$$

69. *(c)* Acidified sodium fusion extract on addition of ferric chloride solution gives blood red colouration, which confirms the presence of N and S.

$$Na + C + N + S \longrightarrow NaCNS \text{ (Sodium thiocyanate)}$$

$$Fe^{3+} + 3CNS^- \longrightarrow \underset{\text{Red ppt.}}{Fe(CNS)_3}$$

70. *(c)* $m = 10$ mg, $v = 100\ ms^{-1}$, $\lambda = ?$

de-Broglie wavelength, $\lambda = \frac{h}{mv}$

$$= \frac{6.63 \times 10^{-34}}{10 \times 10^{-3} \times 100}$$

$$= 6.63 \times 10^{-34}\ \text{m}$$

71. *(b)* Number of electrons in Mg = 12

Number of electrons in $Mg^{2+} = 10$

Number of electrons in $Na^+ = 10$

$\therefore$ Mg^{2+} is isoelectronic with Na^+.

72. *(b)* Gram molecular volume of oxygen at STP is 22400 cm^3.

i.e. volume of 1 mole of oxygen molecule

$$= \frac{22400}{1} = 22400\ cm^3.$$

73. *(a)* Presence of halogen in organic compounds can be detected using Beilstien's test. In this test, an organic compound react with copper wire, when they are heated in a flame and produce a brilliant, long lasting green flame.

74. *(c)* Electronic configuration of Cr = [Ar] $3d^5\,4s^1$

Electronic configuration of Cr^{3+} = [Ar] $3d^3\,4s^0$

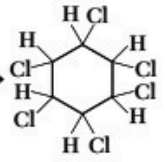
75. *(a)* Equivalent of metal = equivalent of oxygen

$$\frac{w}{31.75} = \frac{8}{8}$$

$$\therefore \quad w = 31.75\ \text{g}$$

76. *(c)* Benzene reacts with chlorine in sunlight to give a final product gammaxene.

$$C_6H_6 + 3Cl_2 \xrightarrow{\text{Sunlight}} C_6H_6Cl_6$$



(Gammaxene)

77. *(c)* In the periodic table metals usually used as catalyst belong to *d*-block as they have variable oxidation state.

78. *(a)* Dalton's law is applicable to mixture of gases, when they are non-reacting, so that the total pressure of all gases is equal to the sum of the partial pressure of each gas. CO and H_2 are non-reacting gases.

$$\therefore \quad p_T = p_{CO} + p_{H_2}$$

79. *(b)* The general formula of cycloalkene is C_nH_{2n-2}.

80. *(c)* In acetylene, between carbon atoms, there are one σ and two π-bonds i.e.

$$\underset{\text{Acetylene}}{H-\overset{\sigma}{\underset{\pi\,\pi}{C\equiv C}}-H}$$

81. *(d)* Denaturated alcohol is ethanol + methanol. Ethanol is made unfit for drinking by adding poisonous substance such as methanol. It is called denaturation.

82. *(a)* During the formation of a chemical bond energy of system decreases with the release of energy. This result in stability of bond formed.

83. *(b)* O_2 is diatomic and degree of freedom is 5.

$$\therefore \text{ Kinetic energy of } O_2 = \frac{5}{2} \times k \times T$$

$$= \frac{5}{2} \times 273 \times k$$

SO_2 is triatomic and degree of freedom is 7.

$$\therefore \text{ Kinetic energy of } SO_2 = \frac{7}{2} \times 546 \times k$$

∴ So, kinetic energy of SO_2 is more than that of O_2 i.e.

$$SO_2 > O_2$$

84. *(d)* + *I*-effect is shown by $-CH_3$ group.

While, $-NO_2$, —Br and —Cl shows −*I*-effect.

85. *(b)* Formation of coloured solution is possible, when metal ion in the compound contains unpaired electrons due to transition of electrons in excited state.

86. *(d)* Temperature, surface tension and viscosity are intensive properties. They are independent of the quantity of matter.

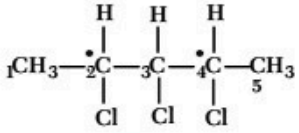
87. *(b)* Hofmann's bromamide reaction is used to convert amide to amine.

$$\underset{\text{Amide}}{R-\overset{O}{\overset{\|}{C}}-NH_2} \xrightarrow{Br_2 + 4NaOH} \underset{\text{Amine}}{R-NH_2} + Na_2CO_3 + 2NaBr + 2H_2O$$

88. *(c)* IUPAC name of $Na_3[Co(NO_2)_6]$ is sodium hexanitro cobaltate (III).

89. *(a)* Thermodynamic standard condition of temperature and pressure is 273 K (0°C) and 1 atm (760 mm Hg).

90. *(b)* The structure of 2, 3, 4-trichloropentane is



Carbon 2 and 4 are chiral carbons as all the groups are different.

∴ Two chiral carbons are present in 2,3-4-trichloropentane.

91. *(b)* The number of unidentate ligands in the complex ion is called coordination number. Coordination is the number of lone pair of electron shared by ligands with central metal atom.

If all the ligand are unidentate, then number of ligand is equal to coordination number.

92. *(b)* The reaction, $2SO_2(g) + O_2(g) \overset{V_2O_5}{\rightleftharpoons}$ is an example of heterogeneous catalysis as the reactant and catalyst are not present in same phase.

93. *(a)* The amino acid which is not optically active is glycine.

$$\begin{array}{l} CH_2-COOH \\ | \\ NH_2 \end{array}$$

94. *(b)* For a stable molecule the value of bond order must be positive.

∵ Stability is directly proportional to bond order.

95. *(a)* The second order reaction is

$$\underset{\text{Methyl ethanoate}}{CH_3COOCH_3} + NaOH \longrightarrow \underset{\text{Sodium salt of carboxylic acid}}{CH_3COONa} + H_2O$$

96. *(c)* According to Baeyer's strain theory, cyclopentane is highly stable as, it has lowest angle strain. The desired angle for sp^3 hybridised carbon in cyclopentane should be 109°28′ and the actual angle is 108°. Thus the angle strain is minimum.

97. *(c)* Configuration of $O_2^{2-} = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^2$.

∴ Number of antibonding electron pairs is four.

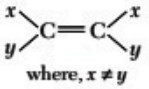
98. *(a)* For 1 M HCl,

$$[H^+] = 1 = 10^0$$

$$[H^+][OH^-] = 10^{-14}$$

$$[OH^-] = 10^{-14} \text{ mol dm}^{-3}.$$

99. *(c)* Geometrical isomerism is shown by alkene with different group attached to carbon i.e.



where, $x \neq y$

100. *(a)* Oxidation state of iron (x) in $K_4[Fe(CN)_6]$ is

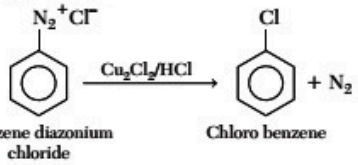
$$4 + x + 6 \times (-1) = 0$$

$$x = 2$$

101. *(c)* During sublimation of naphthalene, phase changes from solid to gaseous state. Hence, the randomness of the particles increases drastically.

$\therefore$ It has maximum increase in entropy.

102. *(c)* Decomposition of benzene diazonium chloride by using Cu_2Cl_2 / HCl to form chlorobenzene is called Sandmeyer reaction.

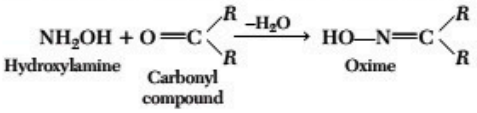


103. *(d)* The complex $[CoCl_3(NH_3)_3]$ cannot ionise in the solution as there is no primary valency in this compound and all the ions are within the coordination entity.

104. *(a)* The reaction $C(s) + O_2(g) \longrightarrow CO_2(g) + 393.5$ kJ is a combustion reaction.

For combustion reaction. ΔH and ΔG are negative but ΔS is positive.

105. *(c)* The product formed, when hydroxylamine condenses with a carbonyl compound is called oxime.



106. *(b)* The electronic configuration of Sc is

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^1 4s^2$

Electronic configuration of Sc^{3+} is $1s^2 2s^2 2p^6 3s^2 3p^6$.

So, Sc^{3+} is colourless due to the absence of unpaired electrons in d-orbital.

107. *(b)* When a sulphur sol is evaporated, sulphur is obtained. On mixing with water, sulphur sol is not formed.

The sol is hydrophobic. Hydrophobic sols are irreversible in nature.

There is a little affinity between the dispersed phase and the dispersion medium.

108. *(d)* When an alkyl halide reacts with alcoholic ammonia in a sealed tube, the product formed will be mixture of primary, secondary and tertiary amine.

109. *(c)* When conc. H_2SO_4 is heated with P_2O_5, the acid is converted into sulphur trioxide.

$$H_2SO_4 + P_2O_5 \longrightarrow SO_3 + 2HPO_3$$

110. *(a)* According to second law of thermodynamics, entropy of the universe is continuously increasing.

111. *(d)* KCN is a salt of strong base (KOH) and weak acid (HCN).

$$KCN + H_2O \rightleftharpoons \underset{\text{Strong base}}{KOH} + \underset{\text{Weak acid}}{HCN}$$

$\therefore$ pH of the solution is greater than 7 at 25°C.

112. *(c)* The reagent used in Clemmenson's reduction is Zn - Hg / conc. HCl. It is used for reduction of carbonyl group to alkanes.

113. *(d)* When KBr is dissolved in water, K^+ ions are hydrated due to ionic nature. KBr is soluble in water due to hydration of K^+ ion by H_2O molecule.

114. *(a)* When the mixture of noble gas is cooled in coconut bulb at 173K, the gases that are not absorbed are He and Ne. As they have low critical temperature.

115. *(d)* Let the volume of 10 N HCl used is x litre.

$\therefore$ The volume of 4N HCl used is $(1 - x)$ litre.

$$\therefore \quad 7 = \frac{10 \times x + 4 \times (10 - x)}{1}$$

$$\therefore \quad x = 0.5 \text{ L}$$

$\therefore$ Volume of 10 N HCl and 4 N HCl used is 0.5 L each.

116. *(c)* The metal present in insulin is zinc.

117. *(b)* The equivalent mass of oxygen in CO and CO_2 remains constant. While, that of carbon changes as it have different oxidation state in different compound.

118. *(d)* Maximum number of molecules of CH_3I, that can react with CH_3NH_2 is 3. The reaction is as follows :

$$CH_3—NH_2 \xrightarrow{CH_3I} (CH_3)_2NH \xrightarrow{CH_3I}$$

$$(CH_3)_3N \xrightarrow{CH_3I} \underset{\text{Tetramethyl ammonium iodide}}{(CH_3)_4\overset{+}{N}I^-}$$

119. *(b)* Ellingham diagram represent a graph of ΔG° *vs* T. It shows the temperature dependence of the stability of compound.

120. *(b)* The ore not containing iron is carnallite.

It's formula is $KCl \cdot MgCl_2 \cdot 6H_2O$.

Mathematics

121. *(d)* We have,

$7^{2\log_7 5}$

$= 7^{\log_7 5^2}$ $[\because m\log a = \log a^m]$

$= 5^2$ $[\because a^{\log_a b} = b]$

$= 25$

122. *(a)* The composition table is

$\otimes_{15}$	3	6	9	12
3	9	3	12	6
6	3	6	9	12
9	12	9	6	3
12	6	12	3	9

From the table 6 is the identity element.

123. *(a)* Since, identity element is its own inverse.

So, minimum number of element is 1.

124. *(d)* We have,

$$|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$$

$$\Rightarrow \quad |\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}-\mathbf{b}|^2$$

$$\Rightarrow \quad (\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}+\mathbf{b}) = (\mathbf{a}-\mathbf{b})\cdot(\mathbf{a}-\mathbf{b})$$

$$\Rightarrow \quad |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a}\cdot\mathbf{b}$$

$$\Rightarrow \quad 4\mathbf{a}\cdot\mathbf{b} = 0$$

$$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b} = 0$$

$$\Rightarrow \quad \mathbf{a} \perp \mathbf{b}$$

So, angle between $\mathbf{a}$ and $\mathbf{b}$ is $90°$.

125. *(b)* We have,

$$\frac{3x^2+1}{x^2-6x+8} = 3 + \frac{18x-23}{x^2-6x+8}$$

$$= 3 + \frac{18x-23}{(x-4)(x-2)}$$

Again, let $\frac{18x-23}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2}$

$$\Rightarrow \quad 18x - 23 = A(x-2) + B(x-4)$$

Put $x = 4$, we get

$$72 - 23 = 2A \Rightarrow A = \frac{49}{2}$$

Put $x = 2$, we get

$$36 - 23 = -2B$$

$$\Rightarrow \quad B = \frac{-13}{2}$$

So, $\frac{3x^2+1}{x^2-6x+8} = 3 + \frac{49}{2(x-4)} - \frac{13}{2(x-2)}$

126. *(c)* We know that, any vector perpendicular to $\mathbf{a}$ and in the plane containing $\mathbf{b}$ and $\mathbf{c}$ is $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$.

Now, $\mathbf{b}\times\mathbf{c} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 2 & -5 \\ 3 & 5 & -1 \end{vmatrix} = 23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Again, $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 23 & -14 & -1 \end{vmatrix}$

$$= -17\hat{\mathbf{i}} - 21\hat{\mathbf{j}} - 97\hat{\mathbf{k}}$$

which is required vector.

127. *(a)* We have,

$|\mathbf{OA}| = 5, |\mathbf{OB}| = 6, \angle\mathbf{BOA} = 60°$

Now, $\mathbf{OA}\cdot\mathbf{OB} = |\mathbf{OA}|\,|\mathbf{OB}|\cos(\angle BOA)$

$$= 5 \times 6\cos 60° = 5 \times 6 \times \frac{1}{2} = 15$$

128. *(a)* We have,

$A(1, -1, 2)$, $B(2, 0, -1)$ and $C(0, 2, 1)$

$\therefore \quad \mathbf{AB} = (2-1)\hat{\mathbf{i}} + (0-(-1))\hat{\mathbf{j}} + (-1-2)\hat{\mathbf{k}}$

$= \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

and $\mathbf{AC} = (0-1)\hat{\mathbf{i}} + (2-(-1))\hat{\mathbf{j}} + (1-2)\hat{\mathbf{k}}$

$= -\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$

Now, required vector $= \mathbf{AB}\times\mathbf{AC}$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$$= 8\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

129. *(d)* Let $S_n = \frac{1}{2\times5} + \frac{1}{5\times8} + \frac{1}{8\times11} + \ldots + \frac{1}{(3n-1)(3n+2)}$

$$= \frac{1}{3}\left[\frac{3}{2\times5} + \frac{3}{5\times8} + \frac{3}{8\times11} + \ldots + \frac{3}{(3n-1)(3n+2)}\right]$$

$$= \frac{1}{3}\left[\frac{5-2}{2\times5} + \frac{8-5}{5\times8} + \frac{11-8}{8\times11} + \ldots + \frac{(3n+2)-(3n-1)}{(3n-1)(3n+2)}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \ldots + \frac{1}{3n-1} - \frac{1}{3n+2}\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right] = \frac{1}{3}\left[\frac{3n+2-2}{2(3n+2)}\right]$$

$$= \frac{n}{2(3n+2)} = \frac{n}{6n+4}$$

130. *(c)* $T_9 = T_{8+1}$

$$= {}^8C_8(3x)^{8-8}\left(\frac{-1}{2x}\right)^8$$

$$= {}^8C_8(3x)^0\left(\frac{-1}{2x}\right)^8$$

$$= 1 \times 1 \times \frac{1}{2^8x^8} = \frac{1}{256x^8}$$

131. *(d)* We have,

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

Since, B is inverse of A, then

$AB = I$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$\therefore 5 - \alpha = 0$ or $\alpha - 5 = 0$ or $5 + \alpha = 10$

$\Rightarrow \quad \alpha = 5$

132. *(b)* Since, A is singular matrix, then

$$|A| = 0$$

$$\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0$$

$\Rightarrow -x[x^2 - 144] = 0 \quad \Rightarrow \quad x = 0, \pm 12$

133. *(d)* We have,

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$\therefore \quad |A| = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$= 1[8-6] + 3[6-4] = 2 + 6 = 8$$

Now, we know that

$$A \cdot (\text{adj}A) = 8I = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

134. *(d)* Since, $f(x) = |x|$ is neither one-one nor onto, so $f(x)$ is not invertible.

$\therefore f^{-1}(x)$ does not exist.

135. *(a)* We have,

$$\begin{bmatrix} x & p & q \\ p & x & q \\ p & q & x \end{bmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} x+p+q & p & q \\ x+p+q & x & q \\ x+p+q & q & x \end{vmatrix}$$

Taking common $(x + p + q)$ from C_1, we get

$$(x+p+q)\begin{vmatrix} 1 & p & q \\ 1 & x & q \\ 1 & q & x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, we get

$$(x+p+q)\begin{vmatrix} 0 & p-x & 0 \\ 0 & x-q & q-x \\ 1 & q & x \end{vmatrix}$$

$$= (x+p+q)\cdot 1\,[(p-x)(q-x) - 0]$$

$$= (x-p)(x-q)(x+p+q)$$

136. *(c)* We have,

Equation of circle is $x^2 + y^2 = 4$. Its centre is $C_1(0, 0)$ and radius (r_1) is 2.

Equation of another circle is

$$x^2 + y^2 - 6x - 8y - 24 = 0$$

i.e. $(x-3)^2 + (y-4)^2 = (7)^2$. Its centre is $C_2(3, 4)$ and radius (r_2) is 7.

Now, $C_1C_2 = \sqrt{(3-0)^2 + (4-0)^2} = 5$

and $|r_1 - r_2| = |2 - 7| = 5$

$\therefore \quad C_1C_2 = |r_1 - r_2|$

So, both circles touch each other internally.

Hence, number of common tangents is 1.

137. *(d)* We have, $3x + y + k = 0$ is tangent to the circle $x^2 + y^2 = 10$.

So, distance of line $3x + y + k = 0$ from centre of the circle will be equal to the radius of circle.

$$\therefore \quad \left|\frac{3(0) + (0) + K}{\sqrt{(3)^2 + (1)^2}}\right| = \sqrt{10}$$

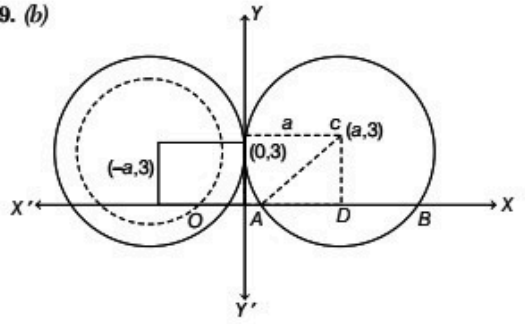
$$\Rightarrow \quad \frac{K}{\sqrt{10}} = \pm\sqrt{10} \quad \Rightarrow K = \pm 10$$

138. *(a)* Negation of $\sim(p \to q) = p \wedge \sim q$

So, negation of given statement is

If 2 is prime and 3 is not odd.

139. *(b)*



Equation of circle will be

$$(x \pm a)^2 + (y-3)^2 = a^2 \quad \ldots(i)$$

Now, in ΔACD,

$$AC^2 = AD^2 + CD^2$$

$$\Rightarrow \quad a^2 = (4)^2 + (3)^2$$

$$[\because AC = a,\ AD = \frac{1}{2}AB = 4,\ CD = 3]$$

$$\Rightarrow \quad a^2 = 25 \quad \Rightarrow a = 5$$

So, equation of circles will be

$$(x \pm 5)^2 + (y-3)^2 = 5^2$$

$$\Rightarrow x^2 \pm 10x + 25 + y^2 - 6y + 9 = 25$$

$$\Rightarrow \quad x^2 + y^2 \pm 10x - 6y + 9 = 0$$

140. *(a)* The triangle formed with vertices $A(0, 0)$, $B\left(0, \frac{3}{2}\right)$ and $C(-5, 0)$ is a right-angled triangle at A. So, BC is hypotenuse.

So, orthocentre of ΔABC is mid-point of BC.

$$\therefore \text{Orthocentre} = \left(\frac{0-5}{2}, \frac{\frac{3}{2}+0}{2}\right) = \left(\frac{-5}{2}, \frac{3}{4}\right)$$

141. *(d)* Let

$$S_1 : x^2 + y^2 - 6x - 6y + 4 = 0$$

$$S_2 : x^2 + y^2 - 2x - 4y + 3 = 0$$

$$S_3 : x^2 + y^2 + 2kx + 2y + 1 = 0$$

Now, radical cnetre of S_1 and S_2 is given by

$$S_1 - S_2 = 0$$

$$\Rightarrow \quad -4x - 2y + 1 = 0$$

$$\Rightarrow \quad 4x + 2y - 1 = 0 \quad \ldots(i)$$

Radical centre of S_2 and S_3 is given by

$$S_2 - S_3 = 0$$

$$-2(1+k)x - 6y + 2 = 0$$

$$(1+k)x + 3y - 1 = 0 \quad \ldots(ii)$$

Now, Eq. (ii) becomes, if

$$k = 1 : 2x + 3y - 1 = 0$$

$$k = 2 : 3x + 3y - 1 = 0$$

$$k = 4 : 5x + 3y - 1 = 0$$

$$k = 5 : 6x + 3y - 1 = 0$$

Now, only $6x + 3y - 1 = 0$ is parallel to $4x + 2y - 1 = 0$ So, for this radical centre does not exists.

So, $k \neq 5$

142. *(b)* We have,

$$S_1 : x^2 + y^2 - 2x - 2y - 7 = 0$$

$$S_2 : x^2 + y^2 + 4x + 2y + k = 0$$

Since, S_1 and S_2 cut orthogonally, then

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

$$\Rightarrow \quad 2(-1)(2) + 2(-1)(1) = -7 + k$$

$$\Rightarrow \quad -4 - 2 = -7 + k$$

$$\Rightarrow \quad k' = 1$$

Now, let r_1 and r_2 be the radius of S_1 and S_2, respectively. Then,

$$r_1 = \sqrt{(-1)^2 + (-1)^2 - (-7)} = 3$$

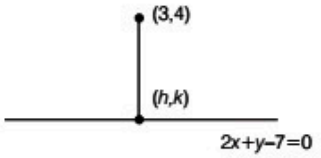
$$r_2 = \sqrt{(2)^2 + (1)^2 - 1} = 2$$

Now, length of common chord

$$= \frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$$

$$= \frac{2 \times 3 \times 2}{\sqrt{(3)^2 + (2)^2}} = \frac{12}{\sqrt{13}}$$

143. *(b)*



Let (h, k) be the coordinate of foot of perpendicular.

Now, slope of line $2x + y - 7 = 0$ is $\left(\frac{-2}{1}\right)$, i.e. -2.

So, slope of perpendicular line.

$$= \frac{-1}{(-2)} = \frac{1}{2}$$

$\therefore$ Equation of perpendicular line is given by

$$y - 4 = \frac{1}{2}(x-3)$$

$$\Rightarrow \quad 2y - 8 = x - 3$$

$$\Rightarrow \quad x - 2y + 5 = 0$$

On solving $2x + y - 7 = 0$ and $x - 2y + 5 = 0$,

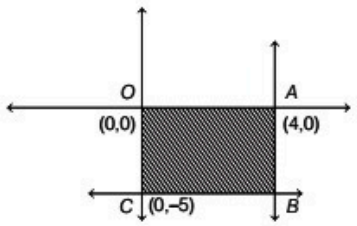
we get $x = \frac{9}{5}$ and $y = \frac{17}{5}$

So, $(h, k) = \left(\frac{9}{5}, \frac{17}{5}\right)$

144. *(b)* We have,

$$xy = 0, x - 4 = 0, y + 5 = 0$$

$$\Rightarrow \quad x = 0, y = 0, x = 4, y = -5$$



So, area enclosed = Area of rectangle $OABC$

$$= AB \times BC$$
$$= 5 \times 4$$
$$= 20 \text{ sq units}$$

145. *(a)* Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then, equation of auxillary circle will be

$$x^2 + y^2 = a^2 \quad (a > b)$$

Now, it is given that

Area of auxillary circle = 2 × Area of ellipse

$$\Rightarrow \quad \pi a^2 = 2 \times \pi ab$$

$$\Rightarrow \quad a = 2b$$

Now, eccentricity of the ellipse,

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{b^2}{(2b)^2}}$$

$$= \sqrt{1 - \frac{b^2}{4b^2}} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

146. *(a)* Since, the number of vertices of odd degree is always even and number of even degree is always even.

So, $m + n$ is an even number.

147. *(c)* We have, equation of ellipse is

$$\frac{x^2}{36} + \frac{y^2}{16} = 1 \Rightarrow \frac{x^2}{(6)^2} + \frac{y^2}{(4)^2} = 1$$

$$\therefore \quad a = 6, b = 4$$

Now, $PS + PS' = 2a = 2 \times 6 = 12$

148. *(d)* We have,

$$\sin\left(2\cos^{-1}\frac{\sqrt{5}}{3}\right) = \sin\left\{\cos^{-1}\left(2 \times \left(\frac{\sqrt{5}}{3}\right)^2 - 1\right)\right\}$$

$$= \sin\left\{\cos^{-1}\left(\frac{10}{9} - 1\right)\right\}$$

$$= \sin\left(\cos^{-1}\frac{1}{9}\right)$$

$$= \sin\left\{\sin^{-1}\sqrt{1 - \left(\frac{1}{9}\right)^2}\right\}$$

$$= \sin\left\{\sin^{-1}\left(\frac{4\sqrt{5}}{9}\right)\right\} = \frac{4\sqrt{5}}{9}$$

149. *(d)* We have,

$$\frac{x^2}{36} - \frac{y^2}{k^2} = 1$$

Option (a) (3, 1)

$$\frac{9}{36} - \frac{1}{k^2} = 1$$

$$\Rightarrow \quad \frac{1}{k^2} = \frac{-27}{36} \quad \Rightarrow \quad k^2 = \frac{-36}{27}$$

which is not possible.

Option (b) (– 3, 1)

$$\frac{9}{36} - \frac{1}{k^2} = 1 \quad \Rightarrow \quad k^2 = \frac{-36}{27}$$

which is not possible.

Option (c) (5, 2)

$$\frac{25}{36} - \frac{4}{k^2} = 1$$

$$\frac{4}{k^2} = \frac{-11}{36}$$

$$k^2 = \frac{-144}{11}$$

which is not possible.

Option (d) (10, 4)

$$\frac{100}{36} - \frac{16}{k^2} = 1 \quad \Rightarrow \quad \frac{16}{k^2} = \frac{64}{36}$$

$\Rightarrow k^2 = 9$, which is possible.

150. *(*)* Because information is not sufficient.

151. *(c)* We have,

$$\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}x + \cot^{-1}x + \cot^{-1}x = \frac{2\pi}{3}$$

$$\Rightarrow \quad \frac{\pi}{2} + \cot^{-1}x = \frac{2\pi}{3}$$

$$\Rightarrow \quad \cot^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow \quad x = \cot\frac{\pi}{6} = \sqrt{3}$$

152. *(a)* We have,

$$\sin^2 17.5° + \sin^2 72.5° = \sin^2 17.5° + \cos^2(90° - 72.5°)$$

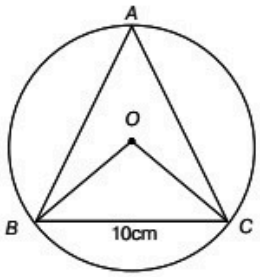
$$= \sin^2 175° + \cos^2 17.5° = 1 = \tan^2 45°$$

153. *(c)* Let $Z = \frac{(1+i)^2}{1-i} = \frac{1+i^2+2i}{1-i} = \frac{2i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{2i+2i^2}{1-i^2} = \frac{2i-2}{1+1} = \frac{2i-2}{2} = -1+i$

$\therefore \quad \overline{Z} = -1-i$

154. *(b)*



We know that,

$\angle BOC = 2\angle BAC = 2 \times 30° = 60°$

Since, $OB = OC$

$\therefore \quad \angle OBC = \angle OCB = \frac{1}{2}(180° - \angle BOC) = 60°$

So, ΔBOC is an equilateral triangle.

$\therefore \quad OB = BC = 10$ cm

Therefore, area of circumcircle $= \pi(OB)^2 = \pi(10)^2$

$= 100\,\pi$ sq cm

155. *(a)* We have,

$\sin 3\theta = \sin\theta$

$\Rightarrow \quad 3\sin\theta - 4\sin^3\theta = \sin\theta$

$\Rightarrow \quad 4\sin^3\theta - 2\sin\theta = 0$

$\Rightarrow \quad 2\sin\theta(2\sin^2\theta - 1) = 0$

$\Rightarrow \quad \sin\theta = 0$ or $\sin\theta = \pm\frac{1}{\sqrt{2}}$

If $\sin\theta = 0$ and if $\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = 0, \pi, -\pi$

$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{-5\pi}{4}, \frac{-7\pi}{4}, \frac{-\pi}{4}, \frac{-3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\therefore$ There are nine solutions.

156. *(b)* We have,

$e^{ix} = \cos x + i\sin x$

Put $x = \frac{\pi}{2}$

$\Rightarrow \quad e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$\Rightarrow \quad e^{i\frac{\pi}{2}} = i \Rightarrow e^{i^2\frac{\pi}{2}} = (i)^i \Rightarrow i^i = e^{-\frac{\pi}{2}}$

$\therefore$ Imaginary part of i^i is zero.

157. *(a)* Let $Z = (1+i)^5$

$= [(1+i)^2]^2[1+i]$

$= [1+i^2+2i]^2[1+i] = (2i)^2(1+i)$

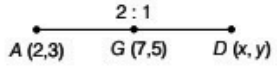
$= -4(1+i) = -4-4i$

Let θ be amplitude of z. Then,

$\theta = \tan^{-1}\left|\frac{-4}{-4}\right| - \pi = \tan^{-1}(1) - \pi$

$= \frac{\pi}{4} - \pi = \frac{-3\pi}{4}$

158. *(a)* We know that, centroid divides median in the ratio 2 : 1 .



$(7,5) = \left(\frac{2x+2}{2+1}, \frac{2y+3}{2+1}\right)$

$(7,5) = \left(\frac{2x+2}{3}, \frac{2y+3}{3}\right)$

$\therefore \quad \frac{2x+2}{3} = 7$ and $\frac{2y+3}{3} = 5$

$\Rightarrow \quad x = \frac{19}{2}$ and $y = 6$

$\therefore \quad D$ is $\left(\frac{19}{2}, 6\right)$.

159. *(b)* We have,

$\lim\limits_{x\to 1} \frac{\tan(x^2-1)}{x-1}$

$= \lim\limits_{x\to 1} \frac{\sec^2(x^2-1)\cdot 2x}{1}$ [using L' Hospital Rule]

$= 2\sec^2 0 = 2$

160. *(c)* We have,

$y = 2^{\log x}$

$\therefore \quad \frac{dy}{dx} = 2^{\log x} \cdot \log 2 \cdot \frac{d}{dx}(\log x) = \frac{2^{\log x}\log 2}{x}$

161. *(b)* We have,

$\sec^{-1}\left(\frac{1+x}{1-y}\right) = a \quad \Rightarrow \quad \frac{1+x}{1-y} = \sec a$

$\Rightarrow \quad \frac{1(1-y) - (1+x)\left(-\frac{dy}{dx}\right)}{(1-y)^2} = 0$

$\Rightarrow \quad 1 - y + (1+x)\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{y-1}{x+1}$

162. *(d)* We have,

$y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2} \Rightarrow y = \cos 3x$

$\Rightarrow \quad \frac{dy}{dx} = -\sin 3x \cdot 3 = -3\sin 3x$

and $\quad \frac{d^2y}{dx^2} = -3\cos 3x \cdot 3 = -9\cos 3x = -9y$

163. *(d)* We have,

$$f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

Since, $f(x)$ is continuous at $x = 0$, then

$$f(0) = \lim_{x \to 0} f(x) \quad \Rightarrow \quad k = \lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$\Rightarrow \quad k = \lim_{x \to 0} \frac{\sin x}{2x}$ [using L' Hospital Rule]

$\Rightarrow \quad k = \frac{1}{2}$

164. *(b)* We have,

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$= (1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)$

$[\because \omega^4 = \omega^3 \cdot \omega = \omega$ and $\omega^8 = (\omega^3)^2 \cdot \omega^2 = \omega^2]$

$= ((1+\omega)(1+\omega^2))^2 = (1+\omega+\omega^2+\omega^3)^2$

$= (\omega^3)^2 \qquad [\because 1+\omega+\omega^2 = 0]$

$= (1)^2 = 1 \qquad [\because \omega^3 = 1]$

165. *(c)* We have,

$$x^x = y^y \quad \Rightarrow \quad x\log x = y\log y$$

$\Rightarrow \quad 1 \cdot \log x + \frac{x}{x} = y'\log y + \frac{y}{y}y'$

$\Rightarrow \quad \log x + 1 = y'(\log y + 1)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1+\log x}{1+\log y}$

166. *(b)* We have,

$$y^2 = x$$

$\Rightarrow \quad 2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

Let the required point be (x_1, y_1). So, slope of tangent

$= \left.\frac{dy}{dx}\right|_{(x_1, y_1)}$

$\Rightarrow \quad m = \frac{1}{2y_1} \qquad \ldots(i)$

Since, tangent makes 45° with X-axis, so

$$m = \tan 45° = 1 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{1}{2y_1} = 1 \quad \Rightarrow y_1 = \frac{1}{2}$$

Again, (x_1, y_1) lies on th curve $y^2 = x$. So,

$$y_1^2 = x_1 \quad \Rightarrow \quad x_1 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

So, required point is $\left(\frac{1}{4}, \frac{1}{2}\right)$.

167. *(d)* We have,

$$x^2y^2 = a^4 \quad \Rightarrow \quad 2xy^2 + x^2(2yy') = 0$$

$\Rightarrow \quad y' = \frac{-2xy^2}{2x^2y} = \frac{-y}{x}$

$\therefore \quad \left.\frac{dy}{dx}\right|_{(-a, a)} = \frac{-a}{-a} = 1$

Now, length of subtangent $= \left|\frac{y}{dy/dx}\right| = \left|\frac{a}{1}\right| = a$

168. *(d)* We have,

$$252 = 2^2 \times 3^2 \times 7^1$$

$\therefore$ Number of positive divisors of

$$252 = (2+1)(2+1)(1+1) = 3 \times 3 \times 2 = 18$$

169. *(b)* We have,

$$5^{124} = (5^3)^{41} \cdot 5$$

Now, $5^3 = 1 \cdot (\text{mod } 124)$

$(5^3)^{41} = 1 \cdot (\text{mod } 124)$

$(5^3)^{41} \cdot 5 = 1 \times 5 \text{ (mod 124)} = 5 \text{ (mod 124)}$

So, the remainder obtained when 5^{124} is divided by 124 is 5.

170. *(a)* **Option (a)** Since, sum of two odd integers is an even integer. Closure property does not hold.

So, it is not a group.

Option (b) Since, sum of two even integers is an even integer.

So, it is a group.

Option (c) $1 \times (-1) = -1$, which belongs to given set.

So, it is a group.

Option (d) $0 + 0 = 0$, which belongs to given set.

So, it is a group.

171. *(b)* We have,

$$y = -x^2 + 6x - 3 \quad \Rightarrow \quad \frac{dy}{dx} = -2x + 6$$

Now, for increasing.

$$\frac{dy}{dx} > 0 \quad \Rightarrow -2x + 6 > 0$$

$\Rightarrow \quad 2x < 6 \quad \Rightarrow x < 3$

So, for $x < 3$, $f(x)$ is increasing.

172. *(c)* Let $I = \int_0^{\pi/2} (\sin^{100} x - \cos^{100} x)\, dx$ ...(i)

$$\Rightarrow \quad I = \int_0^{\pi/2} \left(\sin^{100}\left(\frac{\pi}{2} - x\right) - \cos^{100}\left(\frac{\pi}{2} - x\right)\right) dx$$

$$\left[\because \int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx\right]$$

$$\Rightarrow \quad I = \int_0^{\pi/2} (\cos^{100} x - \sin^{100} x)\, dx \qquad \text{...(ii)}$$

On adding Eqs. (i) and (iii), we get

$$2I = 0 \quad \Rightarrow \quad I = 0$$

173. *(b)* After time t hour distance covered by X and Y are $4t$ and $3t$ respectively.

Now, shortest distance between X and Y after time t hour is given by

$$AB^2 = OA^2 + OB^2 - 2OA \cdot OB \cos\theta$$

$$= (4t)^2 + (3t)^2 - 2 \times 4t \times 3t \cos 120°$$

$$= 16t^2 + 9t^2 - 24t^2 \times \left(\frac{-1}{2}\right)$$

$$= 16t^2 + 9t^2 + 12t^2 = 37t^2$$

$$\Rightarrow \quad AB = \sqrt{37}t$$

Now, $\dfrac{d(AB)}{dt} = \sqrt{37}$

$\therefore$ Rate at which the shortest distance between x and y is increasing is $\sqrt{37}$ km/h.

174. *(b)* Let $I = \int_0^3 t f(t)$

Put $t = 3x \quad \Rightarrow dt = 3dx$

When $t = 0 \Rightarrow x = 0$

and $t = 3 \Rightarrow x = 1$

$$\therefore \quad I = \int_0^1 3xf(3x)(3\, dx)$$

$$= 9\int_0^1 xf(3x)\, dx$$

$$= k\int_0^1 xf(3x)\, dx$$

$$\therefore \quad k = 9$$

175. *(c)* Let $I = \int \dfrac{1}{1 + \cos 8x} dx$

$$= \int \frac{1}{2\cos^2 4x} dx = \frac{1}{2}\int \sec^2 4x\, dx$$

$$= \frac{1}{2}\left(\frac{\tan 4x}{4}\right) + C = \frac{1}{8}\tan 4x + C$$

176. *(a)* Let $I = \int e^x (x^5 + 5x^4 + 1)\, dx$

$$= \int \underset{\text{II}}{e^x} \underset{\text{I}}{(x^5 + 1)}\, dx + \int e^x (5x^4)\, dx$$

$$= (x^5 + 1)\int e^x\, dx - \int\left[\left(\int e^x dx\right)\frac{d}{dx}(x^5 + 1)\right] dx$$

$$+ \int e^x (5x^4)\, dx$$

$$= (x^5 + 1)e^x - \int e^x (5x^4)\, dx + \int e^x (5x^4)\, dx$$

$$= (x^5 + 1)e^x + C = e^x \cdot x^5 + e^x + C$$

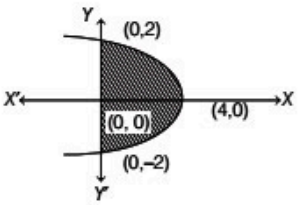
177. *(d)* Let $I = \int \dfrac{x^2 + 1}{x^2 - 1} dx = \int \dfrac{x^2 - 1 + 2}{x^2 - 1} dx$

$$= \int \left(1 + \frac{2}{x^2 - 1}\right) dx = x + 2 \times \frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right| + C$$

$$= x + \log\left|\frac{x - 1}{x + 1}\right| + C$$

178. *(d)* We have,

$$x = 4 - y^2 \quad \Rightarrow \quad y^2 = -(x - 4)$$



Required area $= 2\int_0^4 y\, dx = 2\int_0^4 \sqrt{4 - x}\, dx$

$$= 2\left[\frac{-(4 - x)^{3/2}}{3/2}\right]_0^4$$

$$= \frac{-4}{3}[(4 - 4)^{3/2} - (4 - 0)^{3/2}]$$

$$= \frac{-4}{3}(-8) = \frac{32}{3} \text{ sq units}$$

179. *(b)* We have, equation of line is

$$y = mx + C$$

But $m = C$ (given)

$$\therefore \quad y = Cx + C$$

Now, $\dfrac{dy}{dx} = C$

$$\therefore \quad y = \frac{dy}{dx}(x + 1) \quad \Rightarrow (x + 1)\frac{dy}{dx} - y = 0$$

which is required differential equation.

180. *(c)* We have,

$$\left[1 + \left(\frac{dy}{dx}\right)^5\right]^{1/3} = \frac{d^2y}{dx^2} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^5$$

$\therefore$ Order = 2 and degree = 3